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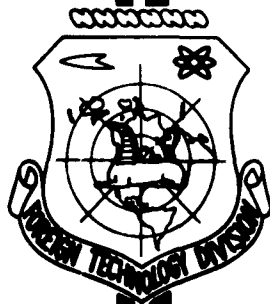
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## TRANSLATION

JOURNAL OF APPLIED MECHANICS AND TECHNICAL PHYSICS  
(SELECTED ARTICLES)

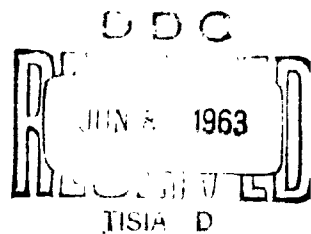
## FOREIGN TECHNOLOGY DIVISION



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# UNEDITED ROUGH DRAFT TRANSLATION

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(SELECTED ARTICLES)

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CONVERSION OF KINETIC ENERGY OF A STATIONARY PLASMA STREAM  
INTO ELECTRICITY WITH ISOTHERMAL RETARDATION  
OF THE STREAM IN A TRANSVERSE MAGNETIC FIELD

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Recently many investigations have been made of the flow of plasma in so-called magnetohydrodynamic generators. In Reference [1] was considered the steady motion of a one-dimensional stream of an incompressible nonviscous liquid through a homogeneous magnetic field in a channel of arbitrary cross section, and that of a compressible nonviscous liquid in a channel of constant cross section at maximum power delivered to the external circuit per unit channel length. In Reference [2] there was considered steady motion of a one-dimensional stream of a compressible nonviscous liquid in a channel of arbitrary cross section through a transverse homogeneous magnetic field with the velocity of the liquid stream and the intensity of the electric field constant along the channel.

Since the maximum temperature that modern materials can withstand and the minimum temperature at which plasma can be produced with conductivity sufficient for effective interaction with a magnetic field are approximately equal, it is of interest to consider the flow of a plasma in a magnetohydrodynamic generator with a plasma temperature constant along the channel.

In the case of isothermal retardation of the plasma, one can use for generation of electricity only the kinetic energy of the plasma, the ratio of which to the total energy of the plasma stream (without

the ionization and dissociation energy) is in the case under consideration

$$\eta = \frac{(\gamma - 1) M^2 / 2\gamma}{(\gamma - 1) M^2 / 2\gamma + 1} \quad \left( \gamma = \frac{c_p}{c_v}, M = \frac{u}{\sqrt{\gamma R T_0}} \right)$$

Here M is the Mach number for isothermal flow. When  $M = 1$  the ratio  $\eta = (\gamma - 1)/(3\gamma - 1)$  for a monatomic gas amounts to approximately 17%.

\* \* \*

1. System of equations. We consider isothermal motion of a one-dimensional stream of compressible nonviscous liquid with finite conductivity in a planar channel of arbitrary rectangular cross section through a transverse magnetic field. The channel walls parallel to the magnetic field are electrodes which make it possible for a potential difference that alternates along the length of the channel to be formed on them (sectionalized electrodes, separated by insulators). The electrodes are interconnected through an external load in such a way that the resistance of the latter is arbitrarily distributed over the length of the channel, while the current in the plasma flows perpendicular to the stream velocity of the plasma and to the magnetic field. We are considering a case when the magnetic field induced by the currents flowing in the plasma can be neglected compared with the external magnetic field.

Let  $\vec{u} = (u, 0, 0)$ ,  $\vec{j} = (0, -j, 0)$ ,  $\vec{H} = (0, 0, H)$ . The motion of the plasma can be described in this case by the following system of equations:

$$\rho u b = \rho_0 u_0 b_0 \quad (\text{continuity equation}) \quad (1.1)$$

$$p = \rho R T_0 \quad (\text{equation of state}) \quad (1.2)$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = -j \frac{H}{c} \quad (\text{equation of motion projected on the channel axis}) \quad (1.3)$$

$$\rho u^2 \frac{du}{dx} = -j^2 \frac{r_e}{\sigma} \quad (\text{energy equation}) \quad (1.4)$$

$$\frac{u''}{\epsilon} = j \left( \frac{r_e}{b} + \frac{1}{\sigma} \right) \quad (\text{equation of the electric circuit}) \quad (1.5)$$

Here  $r_e$  is the resistance of the external circuit per unit area of the walls (the channel electrodes). We shall consider two dependences of the plasma conductivity  $\sigma$  on its density  $\rho$

$$(a) \sigma = \text{const}, \quad (b) \sigma = \sigma_0 \sqrt{\frac{\rho_0}{\rho}} \quad (1.6)$$

The first dependence takes place when the plasma is made of a gas with high ionization potential, with small additives of readily ionized matter, and the plasma temperature and composition are such that the main gas is not ionized but the additives are almost completely ionized [3].

The second dependence takes place in the case of a homogeneous weakly ionized plasma, if the interaction between the electrons and the ions can be neglected compared with the interaction of the electrons and the neutral particles ( $n_e S_{ei} \ll n_n S_{en}$ ).

For the length scale we use the width of the channel  $b_0$  at the section  $x = 0$ ; the scales for the pressure, density, and velocity of the plasma, density of the current, intensity of the magnetic field, conductivity and resistance of the external circuit per unit area of the electrodes we use the values of the following quantities:  $p_0$ ,  $\rho_0$ ,  $u_0$ ,  $j_0$ ,  $H_0$ ,  $\sigma_0$ ,  $r_{e0}$  in the section  $x = 0$ . We shall use dimensionless variables throughout without specially designating them.

2. Optimal retardation of the plasma stream. It follows from (1.4) and (1.5) that the maximum retardation of the plasma and the maximum energy delivery to the external circuit per unit length of the channel occur [1] when

$$r_e = b/\sigma \quad (2.1)$$



Unlike ordinary generators, when the energy released in the internal resistances is parasitic, in a magnetohydrodynamic generator this energy remains in the plasma and can be delivered to an external circuit. (In the case of isothermal retardation, the energy delivered to the external circuit depends only on the values of the initial and final velocities and does not depend on the distribution and value of  $r_e$ .) Therefore the mode (2.1) is very favorable for a magnetohydrodynamic generator.

(A) We consider a case when  $H = 1$  and  $\sigma = 1$ . In this case the system of equations (1.1)-(1.6) is closed. Solving it, we obtain

$$\begin{aligned} i &= \frac{M}{M_0}, \quad p = \rho = \exp \left[ \frac{M^2 - M_0^2}{2} \right] \quad \left( A_0 = \frac{1}{2} \right) \\ b = r_e &= \frac{M_0}{M} \exp \frac{M_0^2 - M^2}{2}, \quad x = \frac{2M_0}{Q_0} \int_M^{M_0} \exp \frac{M^2 - M_0^2}{2} dM \quad (2.2) \\ \frac{db}{dx} &= \frac{Q_0 (M^2 + 1)}{2M^2} \left( M = \frac{u}{\sqrt{RT_0}} \right), \quad A_0 = \frac{c_{j0}}{c_0 u_0 H_0}, \quad Q_0 = \frac{j_0 H_0 b_0}{\rho c} \end{aligned}$$

Here  $M$  is the Mach number for the isothermal flow and  $A_0$  and  $Q_0$  are arbitrary dimensionless parameters.

In the case when  $\sigma = 1/\sqrt{\rho}$  the solution has the form

$$\begin{aligned} p = \rho &= \exp \frac{M^2 - M_0^2}{2}, \quad b = \frac{M_0}{M} \exp \frac{M_0^2 - M^2}{2} \\ i &= \frac{M}{M_0} \exp \frac{M_0^2 - M^2}{4}, \quad r_e = \frac{M_0}{M} \exp \frac{M_0^2 - M^2}{4} \quad (2.3) \\ x &= \frac{2M_0}{Q_0} \int_M^{M_0} \exp \left[ \frac{3}{4} (M^2 - M_0^2) \right] dM \\ \frac{db}{dx} &= \frac{Q_0 (M^2 + 1)}{2M^2} \exp \left[ \frac{5}{2} (M_0^2 - M^2) \right] \end{aligned}$$

(B) Let us consider the problem of the maximum isothermal retardation of the plasma stream with the potential difference on the channel wall-electrodes remaining constant along the channel

$$j r_e = 1 \quad (2.4)$$

For  $\sigma = 1$  the solution has the form

$$i = \frac{M}{M_0} \exp \frac{M^2 - M_0^2}{2} \quad \left( A_0 = \frac{1}{2} \right), \quad x = \frac{2M_0}{Q_0} \int_M^{M_0} \exp \frac{M^2 - M_0^2}{2} dM \quad (2.5)$$

$$p = \rho = H = \frac{M_0}{Mb} = \exp \frac{M^2 - M_0^2}{2}, \quad p = \rho = H \rightarrow \exp \frac{-M_0^2}{2} \text{ as } M \rightarrow 0.$$

The angle of channel expansion will in this case be determined by the formula

$$\frac{db}{dx} = \frac{Q_0}{2} \frac{M^2 + 1}{M^2} \quad \left( \frac{db}{dx} \rightarrow \infty \text{ as } M \rightarrow 0 \right) \quad (2.6)$$

For  $\sigma = 1/\sqrt{\rho}$  the solution has the form

$$i = \frac{M}{M_0} \exp \frac{M^2 - M_0^2}{4} \quad \left( A_0 = \frac{1}{2} \right) \quad (2.7)$$

$$p = \rho = H = \frac{M_0}{Mb} = \exp \frac{M^2 - M_0^2}{2}, \quad x = \frac{2M_0}{Q_0} \int_M^{M_0} \exp \frac{M_0^2 - M^2}{4} dM$$

The angle of channel expansion is determined in this case by the formula

$$\frac{db}{dx} = \frac{Q_0}{2} \frac{M^2 + 1}{M^2} \exp \frac{M_0^2 - M^2}{4} \quad \left( \frac{db}{dx} \rightarrow \infty \text{ as } M \rightarrow 0 \right) \quad (2.8)$$

### 3. Retardation of the plasma in a channel with constant expansion angle.

(A) Let  $H = 1$  and  $\sigma = 1$ . We then get from (1.5)

$$r_0 = \frac{b_0}{\omega_{pe}} \left( \frac{M}{M_0 A_0} - 1 \right) (1 + \chi x) \quad \left( x = 2 \lg \frac{\theta}{2} \right) \quad (3.1)$$

Here  $\theta$  is the angle of channel expansion. For  $j$  we obtain in this case

$$j = \frac{1}{2A_0 M_0} \frac{M}{M^2 - 1} \left\{ -1 + \left[ 1 + 4\chi \frac{A_0}{Q_0} M_0^2 \frac{M^2 - 1}{(1 + \chi x)^2} \right]^{1/2} \right\} \quad (3.2)$$

When  $M \rightarrow 1$  we have  $j \rightarrow \infty$ ; it is therefore impossible to make a continuous transition through the point  $M = 1$ .

When  $M < [1 - Q_0(1 + \chi x)^2 / 4\chi A_0 M_0^2]^{1/2}$  and when  $A_0(M - 1) < 0$  the problem considered has no solution. When  $x = 0$  we have

$$M_0 = \left( \frac{A_0 - 1}{A_0 - \chi / Q_0} \right)^{1/2} \quad (3.3)$$

It follows therefore that the problem has no solution when

$$\min \{1, \chi / |Q_0|\} < |A_0| < \max \{1, \chi / |Q_0|\}$$

Solving the initial system of equations with respect to  $M$ , we ob-

tain

$$\frac{1}{M} \frac{dM}{dx} = \frac{\chi}{(1-\chi^2)(M^2-1)} + \frac{Q_0(1+\chi^2)M^2}{2A_0M_0(M^2-1)^2} \left\{ 1 + \left[ 1 + 4\chi \frac{A_0}{Q_0} M_0^2 \frac{M^2-1}{(1+\chi^2)^2} \right]^{1/2} \right\} \quad (3.4)$$

It follows therefore that  $dM/dx < 0$  only if

$$M < \frac{\sqrt{\chi A_0/Q_0}}{1+\chi^2}$$

In the case when  $\sigma = 1/\sqrt{\rho}$  we have for  $j$  and  $M$

$$j = \frac{1}{2A_0} \left( \frac{M}{M_0} \right)^{1/2} \frac{(1+\chi^2)^{1/2}}{M^2-1} \left\{ -1 + \left[ 1 + 4\chi \frac{A_0}{Q_0} M_0^2 \left( \frac{M_0}{M} \right)^{1/2} \frac{(M^2-1)}{(1+\chi^2)^{1/2}} \right]^{1/2} \right\}$$

$$\frac{1}{M} \frac{dM}{dx} = \frac{\chi}{(M^2-1)(1+\chi^2)} + \frac{Q_0}{2A_0} \left( \frac{M}{M_0} \right)^{1/2} \frac{(1+\chi^2)^{1/2}}{(M^2-1)^2} \left\{ 1 + \left[ 1 + 4\chi \frac{A_0}{Q_0} M_0^2 \left( \frac{M_0}{M} \right)^{1/2} \frac{(M^2-1)}{(1+\chi^2)^{1/2}} \right]^{1/2} \right\} \quad (3.5)$$

(B) Let us consider the case when  $j r_e = 1$  and  $\sigma = 1$ . Solving the system of equations (1.1)-(1.6) and (2.4) we obtain

$$1 + \chi x = \exp \left[ \frac{2\chi A_0}{Q_0(1-A_0)^2} \int_M^{M_0} M^2 \left\{ -1 + \left[ 1 + \frac{4\chi A_0 M^2}{Q_0(1-A_0)^2} \right]^{1/2} \right\} dM \right]$$

$$p = \rho = \frac{M_0}{M(1+\chi^2)}, \quad H = \left[ 1 - \frac{1}{M^2-1} - \frac{\chi}{M(1+\chi^2)} \frac{dM}{dx} \right] \frac{(1-A_0)M_0}{(1+\chi^2)M}$$

$$j = \frac{1}{r_e} = \frac{1}{A_0} \left[ \frac{HM}{M_0} - \frac{1-A_0}{1+\chi^2} \right] \quad \left( \frac{1}{A_0} = 1 + \frac{Q_0 r_e}{b_0} \right) \quad (3.6)$$

The problem has a solution when  $0 < A_0 < 1$ .

Isothermal retardation of the plasma in channels of constant cross section and in narrowing channels ( $\chi \leq 0$ ) is impossible.

4. Retardation of a plasma in a homogeneous magnetic field with the potential difference on the wall-electrodes of the channels constant along the length of the channel. For  $\sigma = 1$  the solution of this problem has the form

$$p = \rho = \frac{1}{1-A_0} \left[ 1 - A_0 \exp \left( \frac{M_0^2 - M^2}{2} \right) \right], \quad b = \frac{M_0}{\rho M}$$

$$j = \frac{1}{r_e} = \frac{M}{M_0 A_0} [1 - (1-A_0)p] \quad \left( \frac{1}{A_0} = 1 + \frac{Q_0 r_e}{b_0} \right)$$

$$x = \frac{M_0}{Q_0(1-A_0)} \int_M^{M_0} \exp \frac{M^2 - M_0^2}{2} dM \quad (4.1)$$

$$\frac{db}{dx} = \frac{(1-A_0)^2 Q_0 \exp [1/2 (M_0^2 - M^2)] (1 + A_0 (M^2 - 1) \exp [1/2 (M_0^2 - M^2)])}{M^2 (1 + A_0 \exp [1/2 (M_0^2 - M^2)])^2}$$

The problem has a solution when  $0 < A_0 < 1$ . If  $\sigma = 1/\sqrt{\rho}$ , then the solution has the form

$$\begin{aligned} p &= p = \left[ 1 - A_0 \exp\left(\frac{M_0^2 - M^2}{2}\right) \right] \frac{1}{1 - A_0}, \quad b = \frac{M_0}{\rho M} \\ i &= \frac{1}{r_0} = \frac{M}{M_0 \sqrt{\rho} A_0} [1 - (1 - A_0) p] \quad \left( \frac{1}{A_0} = 1 + \frac{\omega_{ce}}{\omega_{ci}} \right) \\ z &= \frac{M_0}{Q_0 (1 - A_0)^{3/2}} \int_M^{M_0} \exp \frac{M^2 - M_0^2}{2} \left[ 1 - A_0 \exp \frac{M_0^2 - M^2}{2} \right]^{-1} dM \quad (4.2) \end{aligned}$$

and exists also only in the range  $0 < A_0 < 1$ .

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## GENERATION OF ELECTRIC CURRENT BY A PLASMA FLOWING IN A TUBE

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The simplest generator scheme in the form of a tube of rectangular cross section was first investigated theoretically in [1], where the efficiency of such a generator was estimated in the case of flow with constant conductivity.

It is known that the conductivity of an ionized gas depends exponentially on the temperature, which varies noticeably along the tube. Therefore for a quantitative estimate of the efficiency it is necessary to take into account the temperature dependence of the conductivity.

In the present investigation we estimated the electric efficiency of a generator with conductivity corresponding to a mixture of nitrogen and 1% potassium vapor. In the calculation we used the interpolation formulas obtained by G.S. Aravin and V.P. Shevelev for the dependence of the conductivity on the pressure and the temperature [2]. The results of the calculations are in qualitative agreement with the data of [3], where the flow of cesium vapor was investigated in a magnetohydrodynamic current generator. The presence of friction against the tube walls influences in some cases noticeably the flow of the plasma and will also be accounted for in the analysis of the efficiency.

In solving the problem we disregarded several factors: the anisotropy of the conductivity [7, 8], the terminal losses at the closed points, the heat dissipation of the walls, and finally, the losses to electric resistance in the near-electrode boundary layer. Therefore

the true values of the efficiency are lower for the examples considered in the present paper. We note, however, that to find the most suitable generator mode there are still several possibilities left: variation of the external magnetic field and variation of the cross section of the tube along the length of the channel [9], use of sectionalized electrodes to pick off the Hall current, etc.

\* \* \*

In the case of isotropic conductivity [4], the flow of a plasma in a current generator (Fig. 1) whose horizontal walls are maintained at a constant voltage, and the magnetic permeability and the dielectric constant of the plasma are close to unity, is described by the system of equations

$$\begin{aligned} E_x = E_y = 0, \quad E_z = E_0, \quad \sigma = \sigma(p, \rho), \quad \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0 \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \frac{4\pi}{c} j_z, \quad j_z = \sigma \left( E_0 + \frac{1}{c} (uH_y - vH_x) \right) \quad (1) \\ \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} + \frac{H_y j_z}{c} + \rho \frac{dF_1}{dx} = 0 \\ \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} - \frac{H_x j_z}{c} = 0, \quad w = \frac{\gamma p}{(\gamma - 1)\rho}, \quad j_z r(x) = -E_0 z_0 = k \\ \frac{\partial}{\partial x} \left[ \rho u \left( \frac{u^2 + v^2}{2} + w \right) - \frac{c}{4\pi} E_0 H_y \right] + \\ + \frac{\partial}{\partial y} \left[ \rho v \left( \frac{u^2 + v^2}{2} + w \right) + \frac{c}{4\pi} E_0 H_x \right] = 0 \end{aligned}$$

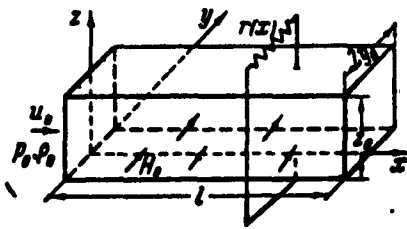


Fig. 1

Here  $\underline{u}$  and  $\underline{v}$  are the velocity components,  $p$  the pressure,  $\rho$  the density,  $r(x)$  the external resistance, and  $dF_1/dx$  denotes the friction force, which depends on the value of the magnetic field, and which is averaged over the cross section.

The boundary conditions for Eqs. (1) will be the equations of continuity on the planes  $\pm y_0/2$  for the components of the magnetic field, and the vanishing of the normal component of the stream velocity. The initial parameters of the stream ( $u_0, p_0, \rho_0$ ) at the section  $x = 0$  can

be specified arbitrarily only in such a range of variation for which the flow out of the tube remains in the sonic mode. In the case of subsonic flow, the parameters at the entrance to the tube are connected with its length  $l$  and with the external pressure  $p_*$ . This connection is obtained by solving the boundary problem. To estimate the various terms in Eqs. (1) we seek the solution in series form

$$\begin{aligned} u &= u_1(x) + u_2(x, y) + \dots, & p &= p_1(x) + p_2(x, y) + \dots, \\ v &= v_1(x, y) + \dots, & \rho &= \rho_1(x) + \rho_2(x, y) + \dots, \\ H_y &= H_0 + H_{2y}(x, y) + \dots, & H_x &= H_{2x}(x, y) + \dots \end{aligned} \quad (2)$$

After solving the equations for  $u_1$ ,  $p_1$ , and  $\rho_1$  we determine the current

$$j_1 = c \left[ E_0 + \frac{u_1 H_0}{c} \right]$$

From the current distribution we can calculate the vortical magnetic field

$$\mathbf{H} = \frac{1}{c} \int \frac{\mathbf{j} \times \mathbf{R}}{R^3} d\omega, \quad \mathbf{R} = (x-x')\mathbf{i}_1 + (y-y')\mathbf{i}_2 + (z-z')\mathbf{i}_3, \quad d\omega = dx'dy'dz' \quad (3)$$

Volume integration shows that  $H_{2x}$  has a maximum in the case of flow between parallel planes ( $z_0 \rightarrow \infty$ )

$$H_{2x} \approx \frac{2j_1 y_0}{c} = R_* H_0, \quad R_* = \frac{4\pi y_0 u_0}{c^2} \quad (4)$$

Taking (4) into account and equating the inertial terms in the equations of motion to the Lorentz forces, we note that

$$u_2 \sim u_1 R_*, \quad p_2 \sim \rho_1 u_1^2 R_*$$

It then follows from the continuity equation that  $v_2 \lesssim u_2 y_0 / l$ . If  $y_0 / l \ll 1$ , then the velocity component  $v_2$  can be neglected in (1) even if account is taken of the influence of the induced field on the main stream. Practical interest attaches to the case when the magnetic Reynolds number  $R_*$  is small. We introduce the dimensionless quantities

$$V = \frac{u}{u_0}, \quad P = \frac{p}{p_0}, \quad Q = \frac{\rho}{\rho_0}, \quad w = \frac{r}{T_0}, \quad F = \frac{F_1}{u_0^2}$$

$$\eta = \frac{\pi H_0^2 \omega}{\rho_0 u_0^2}, \quad \delta = \frac{H_0^2 \omega}{\rho_0 u_0^2}, \quad \kappa = \frac{kc}{u_0 \omega / I_0}, \quad i_0 = \frac{ic}{u_0 \omega / I_0} \quad (5)$$

$$M = u \sqrt{\frac{\rho}{\gamma P}}, \quad \mu = u_0 \sqrt{\frac{\rho_0}{\gamma P_0}}, \quad \sigma_1 = \frac{\sigma}{\sigma_0}$$

Omitting terms proportional to  $R_*$ , we write Eqs. (1) in dimensionless form

$$\frac{dV}{d\eta} + \frac{1}{\gamma \mu^2} \frac{dP}{d\eta} + \sigma_1 |V - \kappa| + Q \frac{dP}{d\eta} = 0, \quad \sigma_1 = \sigma_1(P, W) \quad (6)$$

$$\frac{d}{d\eta} \left[ \frac{V^2}{2} + \frac{PV}{(\gamma-1)\mu^2} \right] + \kappa \sigma_1 |V - \kappa| = 0, \quad P = QW, \quad QV = 1$$

The parameter  $\eta$  represents the ratio of the ponderomotive force to the inertial force  $\rho_0 u_0^2 / x$ ; the parameter  $\kappa$  characterizes the ratio of the voltage drop on the external circuit to the electromotive force in the considered section of the channel, and ranges from zero to unity, when  $\underline{r}$  changes from zero to infinity.

The boundary conditions for (6) are written in the form

$$P = 1, \quad V = 1 \text{ for } \eta = 0$$

$$P = P_* \text{ for } \eta = \delta, \quad \mu_* < 1 \quad (7)$$

$$P = \mu_*^2 V_* \text{ for } \eta = \delta, \quad \mu_* = 1$$

(the asterisk pertains to the quantities at the output from the tube). Analyzing Eqs. (6), we can establish that when  $\mu < 1$

$$\frac{dM}{d\eta} > 0, \quad \frac{dV}{d\eta} > 0, \quad \frac{dP}{d\eta} < 0, \quad \frac{dW}{d\eta} < 0, \quad \frac{dQ}{d\eta} < 0$$

In the case when  $\mu > 1$  the signs of the inequalities are reversed.

Let us estimate now the maximum possible generator efficiency assuming that there is no viscosity and the magnetic Reynolds number is small. In this case Eqs. (6) admit of the important integral

$$P = \frac{C + \mu^2 V - 0.5 \mu^2 V^2}{V / (\gamma - 1) - \kappa / \gamma}, \quad C = \frac{1}{\gamma - 1} + 0.5 \mu^2 - \kappa \left( \mu^2 + \frac{1}{\gamma} \right) \quad (8)$$

Relations (8) are sufficient for an estimate of the efficiency, which we take to mean the ratio of the difference in the energy fluxes at the input and output of the tube to the energy flux at the input.

The expression for the efficiency can be written in the form



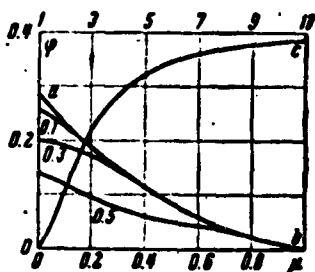


Fig. 2

$$\eta = 1 - \frac{P_* V_* + \frac{1}{2}(\gamma-1)\mu^2 V_*^2}{1 + \frac{1}{2}(\gamma-1)\mu^2} \quad (9)$$

In the case of a sonic outflow mode

$P_* = \mu^2 V_*$  and Expression (9) assumes the form

$$\eta_s = 1 - \frac{\frac{1}{2}(\gamma+1)\mu^2 V_*^2}{1 + \frac{1}{2}(\gamma-1)\mu^2} \quad (10)$$

Let  $\mu < 1$ ; let us find the maximum elec-

tric efficiency for the sonic mode at the out-

put from the tube. Substituting in (8) the expression  $P_* = \mu^2 V_*$  and determining  $V_3$  we obtain

$$V_* = \frac{\gamma-1}{\gamma} \kappa + \frac{1}{\mu} \sqrt{\frac{(\gamma-1)^2 \mu^2 \kappa^2}{\gamma^2} + \frac{2(\gamma-1)}{\gamma+1} C} \quad (11)$$

Differentiating (11) we obtain

$$\frac{\partial \eta_s}{\partial \kappa} < 0, \quad \frac{\partial V_*}{\partial \mu} > 0, \quad \mu < 1$$

Therefore the efficiency tends to its maximum value when  $V_3$  is minimal, i.e., when  $\kappa \rightarrow 1$ . The dependence  $\eta_s(\mu, 1)$  for  $\gamma = 1.4$  is shown in Fig. 2 (curve a-b).

If the external pressure  $P_*$  turns out to be larger than  $\mu^2 V_*$ , then the outflow mode will be subsonic. For each  $0 < P_* < 1$  there exists a  $\mu_0 < 1$  such that when  $\mu < \mu_0$  the outflow mode is subsonic, and when  $\mu > \mu_0$  the outflow mode remains sonic even when  $\kappa = 1$ . From the condition  $\mu_0^2 V_3 = P_*$  we determine

$$\mu_0^2 = \frac{a - \sqrt{a^2 - P_*^2 \gamma^2 (\gamma^2 - 1)}}{\gamma(\gamma - 1)}, \quad a = P_* (\gamma^2 - 1) + 1 \quad (12)$$

The velocity at the output of the tube in the case when  $\mu < \mu_0$  is given by the formula

$$V_* = b + \frac{1}{\mu} \sqrt{b^2 + 2\left(\frac{P_* \kappa}{\gamma} + C\right)}, \quad b = \kappa - \frac{P_*}{(\gamma-1)\mu^2} \quad (13)$$

Since

$$\frac{\partial V_*}{\partial \kappa} < 0 \text{ as } \mu < \sqrt{\frac{(\gamma-1) + (\gamma+1)P_*}{2\gamma}} = \mu_0$$

we have

$$\varphi_{\max} = \frac{(\gamma-1)(1-P_0)}{\gamma} \quad \text{as } \kappa \rightarrow 1, \mu \rightarrow 0 \quad (14)$$

Let us prove that the lines  $\varphi(1, \mu, P_*)$  lie when  $\mu < \mu_0$  below the curve (a-b) on Fig. 2. Assume that there exists a region where  $\varphi > \varphi_3$ . By virtue of the conditions

$$\varphi(1, \mu_0, P_0) = \varphi_3(1, \mu_0), \quad \varphi(1, 0, P_0) < \varphi_3(1, 0)$$

there exists an intersection point  $0 < \mu_2 < \mu_0$ , where

$$\varphi(1, \mu_2, P_0) = \varphi_3(1, \mu_2) = \varphi(1, \mu_2, P_2), \quad P_2 = \mu_2^2 V_3(1, \mu_2)$$

Inasmuch as  $\partial P_2 / \partial \mu_2 > 0$  we have  $P_2 < P_*$ . On the other hand, the derivative  $\delta\varphi / \delta P < 0$  is a monotonic function and consequently the efficiencies cannot be equal at two different points with respect to  $P$  for one and the same  $\mu = \mu_2$ , which proves our statement. The curves  $\varphi(1, \mu, P_*)$  for the cases  $P_* = 0.1, 0.3$ , and  $0.5$  are shown in Fig. 2.

Let us consider now the conditions for the maximum efficiency in the case of supersonic velocity at the inlet to the tube, when two outflow modes are possible: sonic and supersonic. An analysis of Formula (11) shows that there exists a number

$$\kappa_+ = \frac{-a_1 + \sqrt{a_1^2 + 2\gamma(1+1/\gamma)(\gamma-1)\mu^3 a_1}}{a_1} \quad \left( \begin{array}{l} a_1 = (\gamma-1)(\gamma\mu^3+1) \\ a_2 = (\gamma+1)(2-\gamma)\mu^3 \end{array} \right) \quad (15)$$

for which  $V_3 > \kappa$  if  $\kappa < \kappa_+$  and  $V_3 = \kappa_+$  when  $\kappa = \kappa_+$ . From physical considerations  $V \geq \kappa$ . Therefore for  $\kappa > \kappa_+$  only supersonic outflow is possible (the minimum velocity at the output is  $\kappa$ ).

For the sonic outflow mode  $\partial V_3 / \partial \kappa < 0$ , therefore the maximum value of  $\varphi_3$  is attained when  $V_3 = \kappa_+$ . The following inequality holds true

$$\left( \frac{\partial \varphi}{\partial V} \right)_{\kappa, \mu} = \frac{\kappa [P(V) - \mu^2 V]}{\gamma(1+1/\gamma)(\gamma-1)\mu^3} < 0 \quad (\mu > 1)$$

Therefore to determine the maximum efficiency in the case of supersonic outflow it is necessary to put in Formulas (8) and (9)  $V = \kappa$  and obtain the value of  $\kappa_0$  which gives the extremum of  $\varphi$

$$\kappa_0 = \frac{1 + \gamma \mu^2}{(\gamma + 1) \mu^2}, \quad \varphi|_{\kappa=\kappa_0} = (\gamma - 1) \left[ \frac{(1 + \gamma \mu^2)^2}{2(\gamma + 1)(1 + 1/\gamma)(\gamma - 1) \mu^2} - 1 \right] \quad (16)$$

Comparing (15) with (16) we obtain

$$\kappa_0 - \kappa_+ = A [\mu^4 (\gamma - 1)(2 - \gamma^2) + (2 - \gamma)(2\mu^2 - 1)] \quad (A > 0) \quad (17)$$

It follows from (17) that  $\kappa_0 > \kappa_+$  when  $\gamma = 1.4$ , and therefore the value of  $\kappa_0$  which leads to a maximum efficiency equal to  $\varphi(\kappa_0)$  lies in the supersonic region. Figure 2 shows the curve OC calculated from Formula (16) for the case  $\gamma = 1.4$  (the scale is indicated on top of the plot).

Let us estimate the influence of the friction. Following [5], we assume that the friction force averaged over the tube cross section is proportional to the velocity head, i.e.,

$$\frac{dF}{dx} = \xi \frac{u^2}{2y_0}$$

The coefficient  $\xi$ , which depends on the magnetic field [6], on the turbulence of the stream, and on the heat conduction on the wall, will be assumed to be a constant quantity obtained by experiment.

Eliminating the variable  $\eta$  from (6), we arrive at an equation relating the pressure with the velocity

$$\begin{aligned} \frac{dP}{dV} = \psi &= - \frac{(c_2 V - \gamma \sigma_1 \kappa) P + (\gamma - 1) \mu^2 (c_2 \mu^2 - 2\gamma \sigma_1 \kappa V + \gamma \sigma_1 \kappa^2)}{c_2 V^2 + V \sigma_1 \kappa (1 - 2\gamma) + \mu^2 \sigma_1 (\gamma - 1)} \\ \frac{d\eta}{dV} &= - \frac{\gamma + \mu^{-2} \psi}{\gamma \sigma_1 (V - \kappa) + \alpha V} \quad \left( \sigma_1 = \alpha + \gamma \sigma_1, \quad \alpha = - \frac{\gamma P_0 \mu^2 \xi}{2\mu \sigma_1 H_0^2} \right) \end{aligned} \quad (18)$$

If  $\sigma_1$  is constant, then Eqs. (18) can be integrated in quadratures. In this case, however, it is possible to obtain a simple expression for the efficiency only when  $\alpha$  is small. Thus, for example, for  $\mu \rightarrow 0$ ,  $\kappa = 1$ , using the perturbation method, we obtain

$$\varphi(0, 1) = \frac{\gamma - 1}{\gamma} \left[ 1 - P_0 + \alpha \ln \frac{\gamma \mu^2}{1 - P_0} + \frac{\gamma - 1}{\gamma} (1 - P_0) \alpha \right] \quad (19)$$

It follows from (19) that the friction must be taken into account even for small  $\alpha$ , since its influence on the efficiency is proportional to  $\alpha \ln \alpha$ . It is possible to estimate the influence of the viscosity

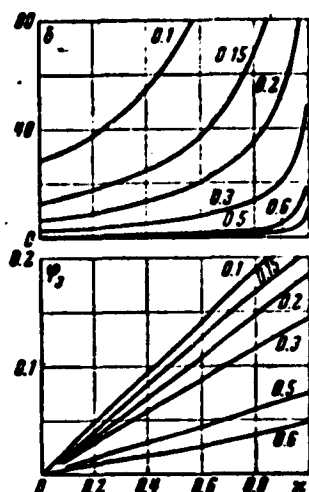


Fig. 3

more rigorously using as an example the flow of an incompressible plasma between parallel planes ( $z_0 \rightarrow \infty$ ) in an external magnetic field [4] (the Hartmann problem). The expression for the efficiency  $\varphi_-$  in the maximum-power mode, which in this case must be taken to mean the ratio of the work done in the external circuit

$$\int_{-0.5a}^{0.5a} \int_{-0.5a}^{0.5a} r(x) j_z^2 dy dz$$

to the pushing work

$$\int_{-0.5a}^{0.5a} u_0 u_0 (p_0 - p_1) dy$$

has the form

$$\varphi_- = \frac{\epsilon \operatorname{ch} \epsilon - \operatorname{sh} \epsilon}{2(\epsilon \operatorname{ch} \epsilon + \operatorname{sh} \epsilon)}, \quad \epsilon = \frac{y_0 H_0}{2c} \sqrt{\frac{c_0}{\lambda}} \quad (20)$$

Here  $\lambda$  is the viscosity coefficient.

In order to calculate the maximum efficiency for a given generator length ( $\delta = \text{const}$ ) it is necessary to integrate Eqs. (18) numerically. It was assumed in the calculations that the plasma conductivity corresponds to thermal ionization of a mixture of nitrogen with 1% potassium vapor and is represented, in accordance with the calculations of [2], in the form

$$\begin{aligned} \sigma &= \sigma_0 \sqrt{\frac{p_1}{p}} \exp \frac{-a}{T} = \frac{\sigma_0}{\sqrt{p}} \exp \left[ b \left( 1 - \frac{1}{\sqrt{p}} \right) \right] \\ \sigma_0 &= \sigma_0 \sqrt{\frac{p_1}{p}} e^{-b}, \quad b = \frac{a}{T_0}, \quad p_1 = 1 \text{ atm} \\ \sigma &= 2.6 \cdot 10^4 \text{ }^\circ\text{K}, \quad \sigma_0 = 9.05 \cdot 10^3 \text{ 1/ohm-cm.} \end{aligned} \quad (21)$$

For subsonic entrance velocities, the parameter  $b$  was assumed equal to 8.5, corresponding to an initial temperature in the plasma of approximately 3000°K.

Figure 3 shows the dependence of the efficiency and of the dimensionless tube length  $\delta$ , for which the sonic outflow mode is attained, on the parameter  $\kappa$  at  $\alpha = 0$  and for  $\mu$  varying from 0.1 to 0.6. As  $\kappa \rightarrow 0$  the curves  $\delta = \delta(\kappa, \mu)$  tend to a finite limit, and as  $\kappa \rightarrow 1$  they increase without limit, since the equations (18) have a singularity in the case  $\alpha = 0, \kappa = 1$ .

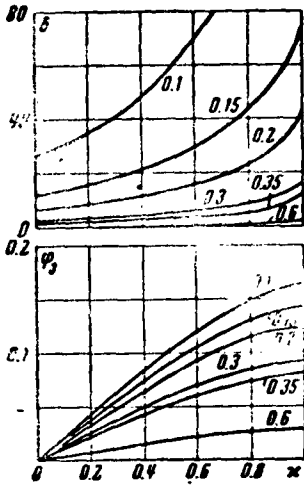


Fig. 4

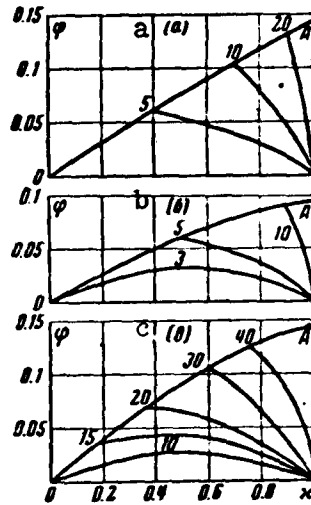


Fig. 5

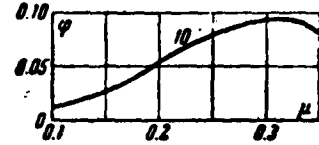


Fig. 6

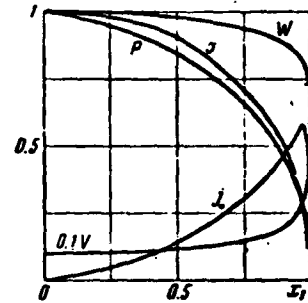


Fig. 7

Figure 4 shows an analogous dependence for the case  $\alpha = 0.1$ . When  $\alpha \neq 0$  Eqs. (18) do not have any singularities and the  $\delta(\kappa, \mu)$  curves tend as  $\kappa \rightarrow 1$  to a finite limit. This is quite obvious, for owing to friction (even in the case of flow without a magnetic field) the sonic mode will be attained in a tube of finite length [5].

Figure 5 shows the dependence of the efficiency on the parameter  $\kappa$  for the following cases, respectively: (a)  $\alpha = 0, \mu = 0.3$ ; (b)  $\alpha = 0.1, \mu = 0.3$ ; and (c)  $\alpha = 0.1, \mu = 0.15$ ; the values of the parameter  $\delta$  are indicated on the curve. From a comparison of Figs. 4 and 5 it follows that the efficiency for constant  $\mu$  increases with increasing  $\delta$ , but always remains smaller than in Fig. 2; the maximum value of the

efficiency for constant  $\mu$  and for constant  $\delta$  is attained for sufficiently large  $\delta$  in the case of a sonic mode at the tube output (curves OA on Fig. 5). For a tube with specified value of the parameter  $\delta = \text{const}$ , there exists a fully defined Mach number at the input to the tube, at which the efficiency is a maximum. This statement is illustrated in Fig. 6, which shows the dependence of the maximum efficiency on the number  $\mu$  for the case  $\delta = 10$  and  $\alpha = 0.1$ .

Figure 7 shows the distribution of the plasma parameters along the length of the tube  $x_1 = x/l$  for the case  $\mu = 0.2$ ,  $\alpha = 0.1$ ,  $\kappa = 1$ . It is seen from the plot that the conductivity changes by one order of magnitude along the tube. In order to compare the efficiency of the generator with subsonic and supersonic velocity at the input to the tube, Figs. 8 and 9 show the dependence of  $\varphi$  and  $\delta_2$  on  $\kappa$  for the cases  $1.5 \leq \mu_3 \leq 4$ ,  $\alpha = 0$  and also the dependence of the efficiency on  $\kappa$  for  $\mu_3 = 2$  and different values of  $\delta_2$ . It was assumed in the calculation that the slowing-down temperature at the inlet of the tube is approximately 3000°K, and therefore the initial values of  $\sigma_{20}$  turn out to depend on the Mach number

$$\begin{aligned} c_1 &= \frac{c_0}{\sqrt{\gamma}} \exp \left[ b_1 \left( 1 - \frac{1}{\sqrt{\gamma}} \right) \right], \quad b_1 = 8.5 \left( 1 + \frac{1-\gamma}{2} \mu_3^2 \right) \\ c_{10} &= c_0 \left( \frac{1 + \frac{1}{2}(\gamma-1)\mu_3^2}{1 + \frac{1}{2}(\gamma-1)\mu^2} \right)^{\frac{\gamma}{2(\gamma-1)}} \exp \left( \frac{1-\gamma}{2} b \mu_3^2 \right) \\ \delta_1 &= \frac{I_{20} l^2}{p_2 \mu_3^2} = \delta \exp \left( \frac{1-\gamma}{2} b \mu_3^2 \right) \left( \frac{1 + \frac{1}{2}(\gamma-1)\mu^2}{1 + \frac{1}{2}(\gamma-1)\mu_3^2} \right)^{\frac{1+\gamma}{2(\gamma-1)}} \end{aligned} \quad (22)$$

The dashed curves on Fig. 8 correspond to values  $\kappa = \kappa_+$ . To the left of  $\kappa = \kappa_+$  the  $\delta_2(\mu_3)$  curves tend to a finite limit. Between the dashed curve and the curve a-b, corresponding to Eq. (16), we have  $\delta_2 \rightarrow \infty$ . However, if friction is taken into account, then  $\delta_2$  will always be finite. In the case of a supersonic flow out of the tube, a complicated system of shocks arises. We note that the pressure which would occur in the case of slowing down of the escaping stream in a

direct shock wave should be larger than the external pressure. If this is not satisfied, then a shock wave is produced in the middle of the tube in such a place that the internal and external pressures become equal at the output of the tube.

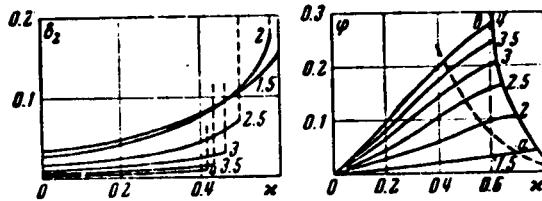


Fig. 8

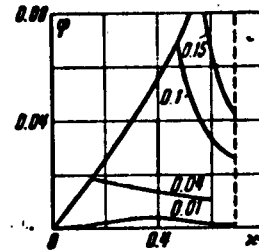


Fig. 9

It follows from Fig. 8 that for large  $\mu$  and for  $\delta \rightarrow \infty$  the efficiency of a supersonic generator becomes higher than in the case when  $\mu < 1$ . On the other hand, if we compare generators of equal length for the same slowing-down parameters, then we have from (22)  $\delta_2 \ll \delta$ ; therefore in the cases of practical interest the subsonic modes are more advantageous than the supersonic ones. Calculations have confirmed the need for taking into account the dependence of the conductivity on the temperature and on the pressure. Thus, for the case  $\delta = 10$ ,  $\mu \approx 0.3$ ,  $\alpha = 0.1$  the maximum of the electric efficiency is  $\approx 10\%$ , and in the case of flow with constant conductivity without friction  $\varphi_{\max} \approx 17\%$ .

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THERMAL IONIZATION AND ELECTRIC CONDUCTIVITY  
OF SOME COMBUSTION MIXTURES AND PRODUCTS

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On the basis of the thermal nature of ionization (the Saha equation), a calculation is made of the electron concentration and electric conductivity of various inert gases (argon, nitrogen, helium) with additives of the most easily ionized substances (cesium, potassium, sodium) in the temperature range 1500-4000°K, and also of the combustion products of some gaseous highly calorific fuels containing the same additives at atmospheric pressure. The dependence of the electric conductivity of the mixture on the temperature, pressure, type of gas, and amount of additive is analyzed. The calculation is made without account of the influence of the electric and magnetic fields on the concentration of the electrons and on the electric conductivity. The calculation data must be regarded as approximate.

\* \* \*

1. Thermal ionization of a mixture of gases. At sufficiently high temperatures, at which dissociation and ionization of the mixture components takes place, an exact determination of the total composition of neutral and charged mixture components necessitates simultaneous solution of the system of chemical and ionization equilibrium equations. However, as shown by preliminary calculations, at sufficiently low degree of ionization it is permissible to neglect the latter and to solve first the chemical equilibrium system and then, using this as

the final chemical composition of the neutral particles, the equilibrium ionization and ion-production processes are considered independently. We shall be interested henceforth in the region of temperatures and pressures where the mixture components are essentially singly ionized, and there is practically no secondary ionization.

A real mixture consists in the general case of several components capable, on the one hand, of being ionized and, on the other hand, of forming negative ions; therefore in the derivation of the general relations for the thermodynamic-equilibrium electron concentration we shall take into consideration, unlike the preceding investigations [1-6], both ionization and production of negative ions.

Assume that under constant-pressure conditions the equilibrium ionization and recombination of each component A of the type  $\underline{i}$ , comprising the mixture, occurs in accordance with the relation  $A_i \rightleftharpoons A_i^+ + e$ , while the process of formation and decomposition of the negative ions  $B^-$  of type  $\underline{j}$  obeys the relation  $B_j^- \rightleftharpoons B_j + e$ . Then, expressing the equilibrium constants in terms of the partial pressures of the corresponding reagents, we obtain

$$K_i = \frac{p_i^+ p_e}{p_i}, \quad K_j = \frac{p_j p_e}{p_j^-} \quad (1.1)$$

From the matter and charge conservation laws and from the equality of the total pressure of the ionized mixture to the sum of the partial pressures of the components we get

$$p_i^0 = \frac{m}{m_0} (p_i + p_i^+), \quad p_j^0 = \frac{m}{m_0} (p_j + p_j^-) \quad (1.2)$$

$$\sum_i p_i^+ = p_e + \sum_j p_j^-, \quad p = \sum_i (p_i + p_i^+) + \sum_j (p_j + p_j^-) + p_e \quad (1.3)$$

Here  $p_i^0$  and  $p_j^0$  are the initial (prior to the ionization and formation of the negative ions) partial pressures of the ionized and electrically negative components of the mixture,  $m_0$  and  $m$  are the initial and final number of moles of mixture, and  $p_e$  is the partial pressure

TABLE 1

Values of the Ionization Potentials  $V_i$  and the Ratio of the Statistical Weights of the Ground State  $2g_i/g$  for Different Components (the Literature Sources are Indicated in the Square Brackets)

1 Компоненты	Li	K	Na	NO	O <sub>2</sub>	H <sub>2</sub> O	OH	H
$V_i, \text{ev}$ 3	3.883 [13]	4.339 [13]	5.139 [13]	9.250 [14]	12.075 [15]	12.500 [15]	13.180 [16]	13.585 [13]
$2g_i/g$	1 [19]	1 [19]	1 [19]	— [19]	8/3 [20, 21]	4 [20, 21]	3/2 [20]	1 [19]
1 Компоненты	O	CO <sub>2</sub>	CO	H <sub>2</sub>	N <sub>2</sub>	Ar	Ne	
$V_i, \text{ev}$ 3	13.614 [13]	13.790 [15]	14.010 [15]	15.427 [17]	15.600 [18]	15.755 [13]	24.580 [13]	
$2g_i/g$	8/9 [19]	2 [19]	Опр- сущ- ность [20]	4 [20]	4 [20]	12 [19]	4 [19]	

1) Component; 2) tentative; 3) ev.

of the electrons.

Solution of this system of equations for the partial pressure of the electrons yields the following expression

$$p_e = \frac{p - p_e}{p} \left( \sum_i \frac{K_i p_i^0}{K_i + p_e} - \sum_j \frac{p_j^0 p_e}{K_j + p_e} \right) \quad (1.4)$$

Neglecting here the value of  $p_e$  compared with  $p$  ( $p_e \ll p$ ) we have

$$p_e = \sum_i \frac{K_i p_i^0}{K_i + p_e} - \sum_j \frac{p_j^0 p_e}{K_j + p_e} \quad (1.5)$$

We note that this assumption is equivalent to considering the indicated processes at constant volume.

Calculation has shown that in the case of high ionization potentials of the mixture (for example, in combustion products without additives of easily ionized substances, Tables 1 and 2) the equilibrium constants  $K_i$  of the ionization and recombination processes are many orders of magnitude smaller than the partial pressure of the electrons  $p_e$ , and the latter is in turn many orders of magnitude smaller than

the equilibrium constants  $K_j$  of the formation and decomposition of the negative ions, owing to the small values of the electron-affinity energy (0.87-1.735 ev). Taking this into account, we obtain a relatively simple relation for the determination of the partial pressure of the electrons at a low degree of ionization

$$p_e = \left[ \sum_i K_i p_i^0 / \left( 1 + \sum_j \frac{p_j^0}{K_j} \right) \right]^{1/2} \quad (1.6)$$

which in the case when no negative ions are produced assumes the well-known form [4]

$$p_e = \left( \sum_i K_i p_i^0 \right)^{1/2} \quad (1.7)$$

The necessary equilibrium constants contained in the formulas can be calculated, as is well known, from the Saha equation, which for a monatomic gas at temperatures at which, on the one hand, the electron excitation can be neglected relative to the ground level\* and, on the other hand, the fine structure of the ground state can be disregarded, has the form

$$\lg K = - \frac{5040I}{T} + 2.5 \lg T - 6.48 + \lg \frac{2g_1}{g_2} \quad (1.8)$$

Depending on the process considered,  $K$  denotes here the equilibrium constant of the ionization and recombination processes,  $K_1$ , or the constant for the formation and disintegration of negative ions,  $K_j$  [atm],  $I$  is the ionization potential\*\*  $V_1$  or the electron-affinity energy  $E_j$  [ev], while  $g_1$  and  $g_2$  are the statistical weights of the ground states of the positive ion and the neutral component, or the electro-negative component and the negative ion;  $T$  is the temperature [ $^{\circ}\text{K}$ ].

As applied to molecular components, Eq. (1.8) must be regarded as approximate, for it does not take into account the contributions of the rotational and vibrational degrees of freedom of the molecules.

TABLE 2

Chemical Composition and Temperature  $T^*$  of the Combustion Products in  $^{\circ}\text{K}$  of Different Combustible Mixtures at Pressure  $p = 1$  atm and Initial Temperature  $T = 2980^{\circ}\text{K}$  ( $\beta$  is the Volume Percentage of Fuel in the Initial Mixture)

2	Горючий состав	$\beta$	$T, ^{\circ}\text{K}$	Распределение атомных компонентов, $\text{P}_i$ , атм									
				$\text{P}_{\text{H}_2\text{O}}$	$\text{P}_{\text{O}_2}$	$\text{P}_{\text{H}_2\text{O}}$	$\text{P}_{\text{H}_2}$	$\text{P}_{\text{O}}$	$\text{P}_{\text{CO}_2}$	$\text{P}_{\text{CO}}$	$\text{P}_{\text{H}_2}$	$\text{P}_{\text{H}}$	$\text{P}_{\text{N}_2}$
3	Бензол — воздух	28.6	2387	2.728 · 10 <sup>-3</sup>	5.008 · 10 <sup>-3</sup>	3.24 · 10 <sup>-3</sup>	6.178 · 10 <sup>-3</sup>	1.859 · 10 <sup>-3</sup>	—	—	1.552 · 10 <sup>-3</sup>	0.644	—
4	Бензол — кислород	69.3	3085	—	4.48 · 10 <sup>-3</sup>	3.83 · 10 <sup>-3</sup>	9.50 · 10 <sup>-3</sup>	3.17 · 10 <sup>-3</sup>	—	—	1.37 · 10 <sup>-3</sup>	—	—
5	Ацетилен — воздух	9.3	2575	3.270 · 10 <sup>-3</sup>	3.71 · 10 <sup>-3</sup>	8.82 · 10 <sup>-3</sup>	5.74 · 10 <sup>-3</sup>	3.79 · 10 <sup>-3</sup>	9.72 · 10 <sup>-3</sup>	3.56 · 10 <sup>-3</sup>	1.22 · 10 <sup>-3</sup>	—	—
6	Ацетилен — кислород	28.2	3210	—	2.48 · 10 <sup>-3</sup>	1.19 · 10 <sup>-3</sup>	8.61 · 10 <sup>-3</sup>	4.13 · 10 <sup>-3</sup>	1.57 · 10 <sup>-3</sup>	2.15 · 10 <sup>-3</sup>	2.97 · 10 <sup>-3</sup>	—	—
7	Оксид углерода — кислород (+0.44% воды)	66.7	2985	—	1.84 · 10 <sup>-3</sup>	1.33 · 10 <sup>-3</sup>	9.10 · 10 <sup>-3</sup>	1.94 · 10 <sup>-3</sup>	4.89 · 10 <sup>-3</sup>	3.43 · 10 <sup>-3</sup>	1.35 · 10 <sup>-3</sup>	—	—

1) Partial pressures of the components  $\text{P}_i$ , atm; 2) combustible mixture; 3) hydrogen-air; 4) hydrogen-oxygen; 5) acetylene-air; 6) acetylene-oxygen; 7) carbon monoxide-oxygen (+0.44% water).

However, the error due to this fact is small, since the ratio of the vibrational and rotational partition functions of the molecules and their ions in the range of our temperatures (1500-4000 $^{\circ}\text{K}$ ) is close to unity [8]. Nonetheless, for the ionization process  $\text{NO} \rightleftharpoons \text{NO}^+ + e$ , which makes the principal contribution to the electron concentration in the combustion products of pure mixtures (without additives), and also for the main processes involving the formation and disintegration of negative ions

$\text{OH}^- \rightleftharpoons \text{OH} + e$  and  $\text{O}^- \rightleftharpoons \text{O} + e$  (see below) we shall use henceforth the available more accurately calculated equilibrium constants\* listed in Table 3.

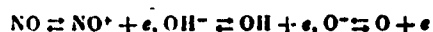
The electron concentration is determined from the formula

$$n_e = p_e n / p \quad (1.9)$$

Here  $p_e$  and  $p$  are, respectively, the partial pressure of the electrons and the over-all pressure of the mixture [atm], and  $n$  is the total concentration

TABLE 3

Equilibrium Constant for the Processes



T°K	$K_{\text{NO}}$	$K_{\text{OH}^-}$	$K_{\text{O}^-}$
2300	$1.764 \cdot 10^{-19}$	0.1253	0.1521
2400	$1.366 \cdot 10^{-19}$	0.2017	0.2304
2500	$9.012 \cdot 10^{-20}$	0.3141	0.3389
2600	$5.162 \cdot 10^{-20}$	0.4746	0.4859
2700	—	0.6981	0.6806
2800	—	1.033	0.9337
2900	—	1.409	1.257
3000	—	1.943	1.665
3100	—	2.632	2.170
3200	—	3.505	2.790

of the molecules at a given pressure and temperature [ $\text{cm}^{-3}$ ].

An analysis of Relations (1.8), (1.5), and (1.9) shows that the concentration of the electrons of a multicomponent mixture is determined essentially by the component having the lowest ionization potential, although it may be present in an insignificant amount. With

further increase in the temperature, when this component turns out to be practically completely ionized and ceases to play a role in the total ionization of the mixture, the electron ionization is determined successively by each following more readily ionized remaining component, up to its complete ionization. The concentration of the negative ions, in contrast to the ionization process, is determined essentially by the component that has the highest electron affinity, and with increase in temperature their concentration decreases. Naturally, the presence of electro-negative components in the mixture, other conditions being equal, reduces the concentration of the electrons owing to their capture by these components.

It is known that in order to increase the electric conductivity of the gases one adds to them small amounts of easily ionized substances, most frequently alkali metals, which have the lowest ionization potentials. It is necessary to note, however, that when alkali metals are added to combustible mixtures it is necessary in the general case to take into account the production of compounds between the alkali metals and certain components of the combustion products and the resultant concentration drop of these metals and hence also the reduction in the electron concentration in the mixture. The most prob-

able are the hydroxides of the alkali metals CsOH [9, 10], KOH [11, 12], NaOH [9] and the lower oxide of potassium  $K_2O$  [11]; the most stable of these at higher temperatures are CsOH and KOH.

Let us estimate approximately the amount of alkali metal A combining with the hydroxyl. We use for the process  $AOH \rightleftharpoons A + OH$  the relations

$$K_{AOH} = \frac{p_A p_{OH}}{p_{AOH}}, \quad p_A^0 = p_A + p_{AOH}$$

and assume that at small amounts of the additives  $p_{OH} \approx p_{OH}^0$ . We then obtain the ratio of the partial pressure of the bound metal (hydroxide)  $p_{AOH}$  to the initial partial pressure of the metal vapor  $p_A^0$

$$\frac{p_{AOH}}{p_A^0} = \frac{p_{OH}^0}{p_{OH}^0 + K_{AOH}} \quad (1.10)$$

According to [9-10] for the process  $CsOH \rightleftharpoons Cs + OH$  and  $KOH \rightleftharpoons K + OH$  at a temperature  $T = 2000^\circ K$  the equilibrium constants are, respectively,  $K_{CsOH} = 3.2 \cdot 10^{-4}$  and  $K_{KOH} = 10^{-3}$  atm, while the thermal effects are 91 and 86 kcal/mole. Extrapolating these constants to the values of the temperatures of the combustible mixtures which we are calculating (Table 2), we obtain from Formula (1.10) that in the combustion products of fuel-oxygen mixtures, owing to their high temperatures, there is practically no cesium or potassium combined with the hydroxyl, while in the combustion products of fuel-air mixtures, which have a lower temperature, not more than 30% cesium and 10% potassium combined with the hydroxyl can be found; to simplify the calculations of the electron concentration and of the electric conductivity, this will be disregarded from now on.

2. Initial data for the calculation of the electron concentration and the electric conductivity of gas mixtures. The values of the ionization potentials and the ratios of the statistical weights of the ground state, necessary for the calculation of the components of dif-



TABLE 4

Values of the Electron Concentration,  $n_e$  in  $\text{cm}^{-3}$ , and the Electric Conductivity  $\sigma$  in  $\text{ohm}^{-1} \text{cm}^{-1}$ , for Inert Gases with 1% by Weight of Cesium, Potassium, and Sodium Additive, and also for Pure Vapors of Alkali Metals at Different Temperatures  $T^\circ\text{K}$  and at a Pressure  $p = 1 \text{ atm}$ .

1 Cases	$T^\circ\text{K}$					
	1500	2000	2500	3000	3500	4000
Ar+1%Cs, $n_e$	$4.15 \cdot 10^{11}$	$1.93 \cdot 10^{12}$	$1.94 \cdot 10^{12}$	$8.65 \cdot 10^{12}$	$2.25 \cdot 10^{13}$	$3.74 \cdot 10^{13}$
	$5.91 \cdot 10^{-3}$	$2.89 \cdot 10^{-1}$	2.18	4.83	7.70	10.20
Ar+1%K $n_e$	$1.36 \cdot 10^{11}$	$9.68 \cdot 10^{11}$	$1.27 \cdot 10^{12}$	$7.04 \cdot 10^{12}$	$2.32 \cdot 10^{13}$	$5.30 \cdot 10^{13}$
	$6.36 \cdot 10^{-4}$	$5.15 \cdot 10^{-2}$	$6.50 \cdot 10^{-1}$	2.63	5.90	9.43
N <sub>2</sub> +1%Cs $n_e$	$3.45 \cdot 10^{11}$	$1.60 \cdot 10^{12}$	$1.60 \cdot 10^{12}$	$7.90 \cdot 10^{12}$	$1.80 \cdot 10^{13}$	$2.83 \cdot 10^{13}$
	$1.19 \cdot 10^{-3}$	$5.57 \cdot 10^{-2}$	$5.06 \cdot 10^{-1}$	1.87	3.86	5.57
N <sub>2</sub> +1%K $n_e$	$1.11 \cdot 10^{11}$	$8.07 \cdot 10^{11}$	$1.06 \cdot 10^{12}$	$5.84 \cdot 10^{12}$	$1.92 \cdot 10^{13}$	$4.29 \cdot 10^{13}$
	$2.86 \cdot 10^{-4}$	$2.19 \cdot 10^{-2}$	$2.79 \cdot 10^{-1}$	1.37	3.55	6.30
N <sub>2</sub> +1%Na $n_e$	$6.71 \cdot 10^9$	$1.04 \cdot 10^{12}$	$2.16 \cdot 10^{12}$	$1.65 \cdot 10^{13}$	$7.01 \cdot 10^{13}$	$2.05 \cdot 10^{14}$
	$1.58 \cdot 10^{-4}$	$2.61 \cdot 10^{-3}$	$5.54 \cdot 10^{-2}$	$4.19 \cdot 10^{-1}$	1.59	3.85
He+1%Cs $n_e$	$1.32 \cdot 10^{11}$	$6.07 \cdot 10^{11}$	$5.98 \cdot 10^{11}$	$2.40 \cdot 10^{12}$	$4.60 \cdot 10^{12}$	$5.20 \cdot 10^{12}$
	$3.84 \cdot 10^{-4}$	$2.02 \cdot 10^{-2}$	$2.12 \cdot 10^{-2}$	$8.40 \cdot 10^{-2}$	1.59	1.93
He+1%K $n_e$	$4.32 \cdot 10^{10}$	$3.08 \cdot 10^{12}$	$3.98 \cdot 10^{12}$	$2.17 \cdot 10^{13}$	$6.41 \cdot 10^{13}$	$9.50 \cdot 10^{13}$
	$1.20 \cdot 10^{-4}$	$9.80 \cdot 10^{-3}$	$1.38 \cdot 10^{-1}$	$7.45 \cdot 10^{-1}$	1.97	2.89
100%Cs $n_e$	$7.58 \cdot 10^{12}$	$3.54 \cdot 10^{14}$	$3.55 \cdot 10^{15}$	$1.68 \cdot 10^{16}$	$4.97 \cdot 10^{16}$	$1.11 \cdot 10^{17}$
	$3.80 \cdot 10^{-4}$	$2.01 \cdot 10^{-2}$	$2.14 \cdot 10^{-1}$	1.08	3.09	6.57
100%K $n_e$	$1.35 \cdot 10^{12}$	$9.59 \cdot 10^{13}$	$1.26 \cdot 10^{14}$	$7.06 \cdot 10^{15}$	$2.40 \cdot 10^{16}$	$6.03 \cdot 10^{16}$
	$6.80 \cdot 10^{-4}$	$5.54 \cdot 10^{-2}$	$8.05 \cdot 10^{-1}$	$4.78 \cdot 10^{-1}$	1.62	3.99
100%Na $n_e$	$6.12 \cdot 10^{10}$	$9.49 \cdot 10^{13}$	$1.97 \cdot 10^{14}$	$1.51 \cdot 10^{15}$	$6.46 \cdot 10^{15}$	$1.93 \cdot 10^{16}$
	$4.10 \cdot 10^{-4}$	$7.34 \cdot 10^{-3}$	$1.70 \cdot 10^{-2}$	$1.40 \cdot 10^{-1}$	$6.18 \cdot 10^{-1}$	1.81

1) Mixture.

ferent mixtures and products of combustion, are chosen in accord with Table 1, from which it is seen that the smallest ionization potentials are possessed by the alkali metals Cs, K, Na, and nitric oxide NO.\* As far as we know, of all the components indicated in the table the largest electron affinity is possessed by OH - 1.735 ev [22], O - 1.465 ev [23, 24], and O<sub>2</sub> - 0.87 ev [25].

The equilibrium chemical composition of the products and the com-

bustion temperature  $T_*$  in  $^{\circ}\text{K}$  of the combustible mixtures employed\* are listed in Table 2.

The electric conductivity of the mixture is determined from the relation [27, 28]

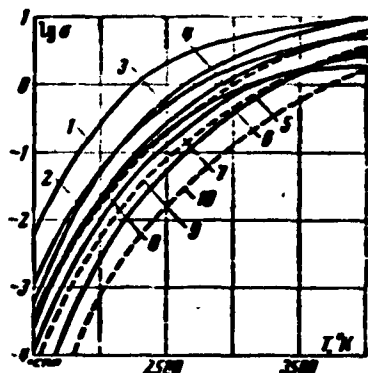


Fig. 1. Dependence of the electric conductivity on the temperature for inert gases (argon, helium, nitrogen) with additives of alkali metals (cesium, potassium, sodium) and pure vapors of alkali metals: 1) Ar + 1% Cs; 2) Ar + 1% K; 3)  $\text{N}_2$  + 1% Cs; 4)  $\text{N}_2$  + 1% K; 5)  $\text{N}_2$  + 1% Na; 6) He + 1% Cs; 7) He + 1% K; 8) 100% Cs; 9) 100% K; 10) 100% Na.

$$\sigma = 0.532 \frac{e^2 n_e}{(m_e k T)^{3/2} \sum_i n_i Q_i} \quad (2.1)$$

where  $e$  and  $m_e$  are, respectively, the charge and mass of the electron,  $n_e$  the electron concentration [ $\text{cm}^{-3}$ ],  $k$  Boltzmann's constant,  $T$  the temperature [ $^{\circ}\text{K}$ ], and  $n_i$  and  $Q_i$  the concentration [ $\text{cm}^{-3}$ ] and the average electronic diffusion collision cross section for neutral particles or ions of the mixture [ $\text{cm}^2$ ]. For ions, the latter is calculated from the formula [28]

$$Q_i = 0.50 \left( \frac{e}{kT} \right)^2 \ln \left[ \frac{1.5}{\sqrt{2} x_i} \left( \frac{kT}{x_i} \right)^{1/2} \right]$$

There are few reliable data on the cross sections for the collisions of electrons with the neutral components, pertaining to the slow electrons with energy less than 0.5 eV which are of interest to us. In the calculation of the electric conductivity of mixtures of argon, helium, and nitrogen with additives of cesium, potassium, and sodium in the temperature interval from 1500 to  $4000^{\circ}\text{K}$ , we assumed the following values of the cross sections: for Ar -  $2 \cdot 10^{-17}$  [29, 30], He -  $6.8 \cdot 10^{-16}$  [31],  $\text{N}_2$  - depending on the temperature  $(5.0-7.4) \cdot 10^{-16}$  [29], Cs and K -  $4 \cdot 10^{-14}$  [32], Na -  $3 \cdot 10^{-14} \text{ cm}^2$  [32].

The cross sections for the collisions between the electrons and

the combustion-product components were taken to be: for H -  $1.5 \cdot 10^{-15}$  [33],  $H_2$  -  $1.2 \cdot 10^{-15}$  [32, 34], O -  $(1.6-2.1) \cdot 10^{-15}$ ,  $O_2$  -  $(3.3-3.6) \times 10^{-16}$ ,  $N_2$  -  $(6.3-6.4) \cdot 10^{-16}$ , NO -  $(5.4-5.6) \cdot 10^{-16}$  [29], CO -  $(7-8) \times 10^{-16}$ ,  $CO_2$  -  $(1.2-3) \cdot 10^{-15}$  [33],  $H_2O$  -  $(5-7) \cdot 10^{-15}$  [33, 34], OH -  $2 \cdot 10^{-15} \text{ cm}^2$  (tentative).

It follows from the foregoing data that the largest collision cross sections are possessed by the alkali metals, and the smallest by argon (the Ramsauer-Townsend effect).

3. Results of the calculations. a. Electron concentration and electric conductivity of inert gases with alkali-metal additives. The calculated dependence of the electron concentration and of the electric conductivity of the mixture on the temperature, pressure, and amount of additive has been determined in the range of variation of practical interest for each of these parameters.

The results of calculation, based on Relations (1.5), (1.9), and (2.1), of the electron concentration and of the electric conductivity of the inert gases argon, helium, and nitrogen\* with additives of cesium and potassium, and in the case of nitrogen also with additive of sodium, in the temperature interval from 1500 to 4000°K at atmospheric pressure and at a constant additive content amounting to 1% of the weight of the mixture are listed in Table 4, while the data for the electric conductivity are plotted in Figs. 1 and 2. For the sake of comparison we show here also the data calculated under these conditions from Formulas (1.4), (1.9), and (2.1) for the electric conductivity of the pure cesium, potassium, and sodium vapor.

It follows from the curves of Fig. 1 that in the entire temperature interval, the largest electric conductivity, reaching a value  $\sigma \approx 10 \text{ ohm}^{-1} \text{ cm}^{-1}$  at a temperature  $T = 4000^\circ\text{K}$ , is possessed by argon with cesium additive. In the central part of the calculated tempera-

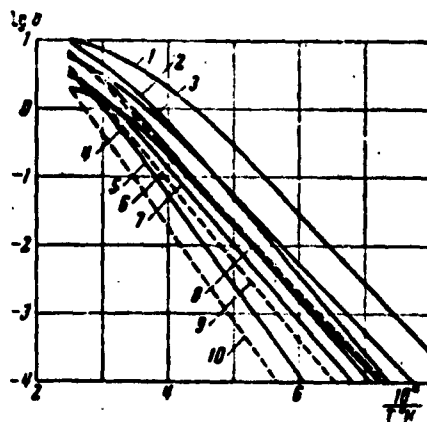


Fig. 2. Dependence of the electric conductivity on the reciprocal temperature for inert gases (argon, helium, nitrogen) with additives of alkali metals (cesium, potassium, sodium) and pure vapors of alkali metals:  
 1) Ar + 1% Cs; 2) Ar + 1% K;  
 3) N<sub>2</sub> + 1% Cs; 4) N<sub>2</sub> + 1% K;  
 5) N<sub>2</sub> + 1% Na; 6) He + 1% Cs;  
 7) He + 1% K; 8) 100% Cs; 9)  
 100% K; 10) 100% Na.

ture interval, approximately between 2000 and 3300°K, the electric conductivity of argon, nitrogen, and helium is successively decreased relative to the value with cesium and potassium additives. The smallest electric conductivity is in this case that of the mixture of nitrogen with sodium.

The electric conductivity of 100% sodium, potassium, and cesium vapor (see the dashed lines on the figures) rise with decreasing ionization potential from sodium to cesium, which is natural, but with the exception of a small region of low and high temperatures, the electric conductivity of 100% cesium vapor turns out to be lower than the electric conductivity of argon and nitrogen with cesium and potassium additives, while the electric conductivity of 100% potassium vapor is lower than the electric conductivity of helium mixed with potassium and cesium, owing to the small mean free path of the electrons due

to the larger cross sections for the collision between the latter and the alkali metals.

As can be seen from Fig. 2, the temperature variation of the electric conductivity at low degree of ionization of the mixture (up to a temperature  $\approx 2500^{\circ}\text{K}$  for the argon-cesium mixture and up to a temperature  $\approx 3000^{\circ}\text{K}$  for the remaining mixtures) can be approximated with sufficient accuracy by an exponential dependence of the type  $\sigma = C \exp(-D/T)$ . With further increase in the temperature, and consequently also in the ionization, the increase in the electric conductivity slows down more and more as the initial substances become consumed and because of the relative decrease in the mean free path of the electrons, owing to the more probable collision between them and the ions (although the electron-ion collision cross section decreases with increasing temperature, nevertheless it is several orders of magnitude larger than the cross section for neutral molecules).

The calculated data, in the same temperature interval, for the dependence of the electron concentration and the electric conductivity of the mixture on the pressure, in its range of variation from 0.01 to 10 atm, using a mixture of nitrogen with 1% potassium by weight as an example, are listed in Table 5 and are plotted for the electric conductivity in Fig. 3. It follows from these data that the electric conductivity of the mixture decreases with increasing pressure, and up to a temperature  $\approx 2500^{\circ}\text{K}$  the increase follows the law  $\sigma\sqrt{p} = \text{const}$ , i.e., it is inversely proportional to the square root of the pressure.

In the general case, for a small degree of ionization (on the order of 3%), we can assume approximately that the dependence of the electric conductivity of the given mixture on the pressure and on the temperature satisfies the equation

$$\sigma = \frac{C}{\sqrt{p}} \exp\left(-\frac{D}{T}\right)$$

in particular, for a mixture of nitrogen with 1% potassium by weight we have

$$\sigma = \frac{9.05 \cdot 10^9}{\sqrt{p}} \exp\left(\frac{-2.6 \cdot 10^4}{T}\right)$$

The dependence of the electric conductivity of the above-mentioned mixtures of inert gases with additives of alkali metals on their weight fraction  $\gamma$  in the mixture, at atmospheric pressure and at a temperature  $T = 3000^\circ\text{K}$ , is shown in Fig. 4.

TABLE 5

Values of the Electron Concentration  $n_e$ , in  $\text{cm}^{-3}$ , and of the Electric Conductivity  $\sigma$ , in  $\text{ohm}^{-1} \text{cm}^{-1}$ , for Nitrogen with 1% Potassium by Weight Added at Different Pressures  $p$  in atm and Temperatures  $T$  in  $^\circ\text{K}$

$T^\circ\text{K}$		$p, \text{Atm}$			
		0.01	0.1	1.0	10
$n_e$	1500	$1.11 \cdot 10^{10}$	$3.57 \cdot 10^{10}$	$1.11 \cdot 10^{11}$	$3.57 \cdot 10^{11}$
	2000	$8.07 \cdot 10^{11}$	$2.62 \cdot 10^{12}$	$8.07 \cdot 10^{12}$	$2.56 \cdot 10^{13}$
	2500	$1.03 \cdot 10^{13}$	$3.35 \cdot 10^{13}$	$1.06 \cdot 10^{14}$	$3.35 \cdot 10^{14}$
	3000	$4.94 \cdot 10^{13}$	$1.78 \cdot 10^{14}$	$5.84 \cdot 10^{14}$	$1.88 \cdot 10^{15}$
	3500	$1.08 \cdot 10^{14}$	$5.21 \cdot 10^{14}$	$1.92 \cdot 10^{15}$	$6.36 \cdot 10^{15}$
	4000	$1.30 \cdot 10^{14}$	$9.65 \cdot 10^{14}$	$4.29 \cdot 10^{15}$	$1.56 \cdot 10^{16}$
$\sigma$	1500	$2.89 \cdot 10^{-3}$	$9.22 \cdot 10^{-3}$	$2.86 \cdot 10^{-2}$	$9.24 \cdot 10^{-2}$
	2000	$2.01 \cdot 10^{-1}$	$6.80 \cdot 10^{-1}$	$2.19 \cdot 10^{-1}$	$7.00 \cdot 10^{-1}$
	2500	1.53	$7.52 \cdot 10^{-1}$	$2.79 \cdot 10^{-1}$	$9.19 \cdot 10^{-1}$
	3000	3.69	2.70	1.37	$5.19 \cdot 10^{-1}$
	3500	5.30	5.20	3.55	1.67
	4000	6.48	7.00	6.30	3.75

1) Atm.

From the curves in this figure, which were calculated from Formula (2.1), we see that for specified conditions with respect to pressure and temperature, a maximum electric conductivity of the gas is attained at a certain (optimal) amount of additive. Further increase of the additive leads to a reduction in the electric conductivity. Qualitatively this follows directly from Formula (2.1), according to which the electric conductivity increases with increasing electron con-

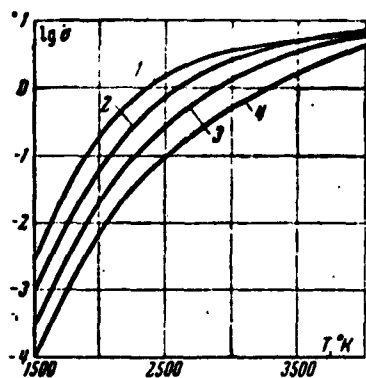


Fig. 3

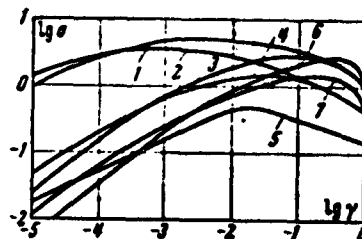


Fig. 4

Fig. 3. Dependence of the electric conductivity of nitrogen + 1% potassium by weight on the temperature at different pressures: 1) 0.01 atm; 2) 0.1 atm; 3) 1 atm; 4) 10 atm.

Fig. 4. Dependence of the electric conductivity of inert gases (argon, helium, nitrogen) with alkali metals added (cesium, potassium, sodium) on the weight fraction of the additives ( $T = 3000^\circ\text{K}$ ,  $p = 1$  atm): 1) Ar + Cs; 2) Ar + K; 3)  $\text{N}_2$  + Cs; 4)  $\text{N}_2$  + K; 5)  $\text{N}_2$  + Na; 6) He + Cs; 7) He + K.

centration and decreases with increasing electronic collision cross section. Naturally, the increase in the amount of impurity in the form of alkali metals, which have much higher cross sections compared with ordinary gases, leads on the one hand to an increase in the electric conductivity due to the increase in the electron concentration in the mixture, and on the other hand to a decrease due to the increase in the total electronic collision cross section:

$$Q = \frac{1}{n} \sum_i n_i Q_i$$

As a net result, at a definite (optimal) amount of impurity, a maximum of electric conductivity sets in, and the dependence of the maximum on various factors can be established by an analysis of the indicated formula.

For the sake of simplicity let us take a two-component mixture, consisting of a fundamental component of some electro-positive gas and

the vapor of some additive, and neglecting the ionization of the fundamental component let us assume that in our temperature range only the additive substance is ionized. Expressing in Formula (2.1) the additive concentration  $n_1$ , the concentration of the main component  $n_2$ , and the electron concentration  $n_e$  in terms of the corresponding partial pressures  $p_1$ ,  $p_2$ , and  $p_e$ , we obtain with the aid of (1.1), (1.3), and (1.9)

$$\dot{\epsilon} = \frac{0.532e^2}{(m_e kT)^{1/2}} \frac{p_e}{(Q_1 - Q_2) p_e^2 / K_1 + (Q_1^2 - 2Q_2) p_e + p Q_2}$$

Differentiating this expression with respect to  $p_e$  under the simplifying assumption that the collision cross section of the electrons with the positive ions of the additive  $Q_1^+$  does not depend on the partial pressure  $p_e$  of the electrons (the error introduced by this in the electric conductivity is insignificant in our case), and equating the first derivative to zero, we obtain a relation for the optimum partial pressure of the electrons:

$$p_{e \text{ opt}} = \left( \frac{p K_1 Q_2}{Q_1 - Q_2} \right)^{1/2}$$

Using this equation along with Eqs. (1.1)-(1.3) for the ionization of the additive, we obtain the optimum ratio  $\alpha_{\text{opt}}$  of the weight of the additive  $G_1$  to the weight of the gas  $G_2$

$$\alpha_{\text{opt}} = \frac{\mu_1 \left[ \frac{Q_2}{Q_1 - Q_2} + \sqrt{\frac{K_1 Q_2}{p(Q_1 - Q_2)}} \right]}{\mu_2 \left[ 1 - 2 \sqrt{\frac{K_1 Q_2}{p(Q_1 - Q_2)}} - \frac{Q_2}{Q_1 - Q_2} \right]} \quad (3.1)$$

where  $\mu_1$  and  $\mu_2$  are, respectively, the molecular weight of the additive and the gas.

If we assume that the collision cross sections are independent of the temperature, then it follows from this formula that the weight ratio  $\alpha_{\text{opt}}$  increases with increasing temperature, owing to the increase in the ionization constant  $K_1$ , and decreases with the increase in the



total pressure  $p$ ,

Neglecting in the numerator of (3.1) the expression under the square root and neglecting in the denominator both terms compared with

unity, we can assume for a clearly tentative estimate under the conditions considered

$$\alpha_{opt} = \frac{n_1}{n_2} \frac{Q_2}{Q_1 - Q_2}$$

which coincides in the particular case when  $Q_2 \ll Q_1$  with the concentration ratio [35]

$$n_1/n_2 = Q_2/Q_1.$$

The values of the optimum weight

fraction of the additive in the mixture

$$\gamma_{opt} = \frac{\alpha_{opt}}{1 + \alpha_{opt}}$$

and of the corresponding maximum electric conductivity  $\sigma_{max}$  at atmospheric pressure and at a temperature  $T = 3000^\circ K$  are listed in Table 6 for various mixtures. It is seen from this table that for a mixture of argon with potassium and cesium, only a very insignificant amount of optimal additive (less than 1%) is necessary to attain maximum electric conductivity of 3.57 and 5.2  $\text{ohm}^{-1} \text{cm}^{-1}$ , respectively; for mixtures of nitrogen with sodium, potassium, and cesium this value does not exceed 8%, and for mixtures of helium with potassium and cesium it reaches 37% at relatively lower absolute values of the maximum electric conductivity.

b. Concentration of electrons and electric conductivity of combustion products. The concentration of the electrons, determined on the basis of the Saha equation (1.8), and the electric conductivity of combustion products without additives of readily ionized substances

TABLE 6

Values of the Optimum Weight Fraction  $\gamma_{opt}$  % and the Maximum Electric Conductivity  $\sigma_{max}$ ,  $\text{ohm}^{-1} \text{cm}^{-1}$ , for Different Mixtures at Pressure  $p = 1 \text{ atm}$  and Temperature  $T = 3000^\circ K$

l Cмечк	$\gamma_{opt}$	$\sigma_{max}$
Ar + Cs	0.22	5.20
Ar + K	0.06	3.57
He + Cs	37.00	2.71
He + K	14.80	1.50
N <sub>2</sub> + Cs	7.95	2.71
N <sub>2</sub> + K	2.42	1.50
N <sub>2</sub> + Na	1.85	0.44

1) Mixture.

are small and according to calculation, with allowance for the data of Table 3, do not exceed\*  $n_e = 1.86 \cdot 10^{10} \text{ cm}^{-3}$  and  $\sigma = 3.53 \cdot 10^{-5} \text{ ohm}^{-1} \text{ cm}^{-1}$ , the latter figures pertaining to the combustion products of acetylene with oxygen. Addition of easily ionized substances, particularly cesium and potassium, in amounts of 1% by weight, as can be seen from Table 7, greatly increases the electron concentration and the electric conductivity of the combustion products.

TABLE 7

Values of the Electron Concentration  $n_e$  in  $\text{cm}^{-3}$  and of the Electric Conductivity  $\sigma$  in  $\text{ohm}^{-1} \text{ cm}^{-1}$  for Combustion Products with 1% by Weight of Cesium, Potassium, and Sodium Added at a Pressure  $p = 1 \text{ atm}$

1 Горючая смесь	T, °K	Cs		K		Na	
		$n_e$	$\sigma$	$n_e$	$\sigma$	$n_e$	$\sigma$
2 Водород — воздух	2387	$9.34 \cdot 10^{13}$	$8.50 \cdot 10^{-1}$	$5.90 \cdot 10^{13}$	$5.13 \cdot 10^{-1}$	$1.10 \cdot 10^{13}$	$9.47 \cdot 10^{-2}$
3 Водород — кислород	3065	$5.62 \cdot 10^{14}$	$3.64 \cdot 10^{-1}$	$4.92 \cdot 10^{14}$	$3.13 \cdot 10^{-1}$	$1.45 \cdot 10^{14}$	$9.44 \cdot 10^{-2}$
4 Ацетилен — воздух	2575	$1.98 \cdot 10^{14}$	$3.52 \cdot 10^{-1}$	$1.36 \cdot 10^{14}$	$2.19 \cdot 10^{-1}$	$2.93 \cdot 10^{13}$	$4.69 \cdot 10^{-2}$
5 Ацетилен — кислород	3210	$1.04 \cdot 10^{15}$	1.50	$9.55 \cdot 10^{14}$	1.27	$3.02 \cdot 10^{14}$	$4.34 \cdot 10^{-1}$
6 Окись углерода — кислород	2945	$7.18 \cdot 10^{14}$	1.29	$5.81 \cdot 10^{14}$	1.02	$1.62 \cdot 10^{14}$	$2.97 \cdot 10^{-1}$

1) Combustible mixture; 2) hydrogen-air; 3) hydrogen-oxygen; 4) acetylene-air; 5) acetylene-oxygen; 6) carbon monoxide-oxygen.

The greatest value of the electron concentration and the electric conductivity is possessed in this case (Table 7) by the combustion products of acetylene and of carbon monoxide with oxygen. It must be noted that in spite of the fact that the temperature of the combustion products of hydrogen with oxygen is  $100^\circ$  higher than the temperature for carbon monoxide with oxygen, however, for an almost equal electron concentration, their electric conductivity (owing to the larger value of the collision cross section  $Q_*$  of the combustion products (Table 8) of this mixture the main contribution is made by the water) is several times smaller than the electric conductivity of the combustion products

TABLE 8

Values of Optimal Content  $\gamma_{\text{opt}}$  % of Additives of Cesium, Potassium, and Sodium and of the Maximum Electric Conductivity  $\sigma_{\text{max}}$  in  $\text{ohm}^{-1} \text{cm}^{-1}$  for the Combustion Products of Combustible Mixtures with Alkali-Metal Additives

1 Горючая смесь	$T, ^\circ\text{K}$	$P_0$	$Q_{\text{сж}}, \text{cal}^*$	Cs		K		Na	
				$\gamma_{\text{opt}}$	$\sigma_{\text{max}}$	$\gamma_{\text{opt}}$	$\sigma_{\text{max}}$	$\gamma_{\text{opt}}$	$\sigma_{\text{max}}$
2 Водород - воздух	2387	24.3	2.7	30.0	0.27	11.0	0.10	9.5	0.0
3 Водород - кислород	3065	14.6	4.2	54.0	1.83	26.7	0.79	24.0	0.29
4 Ацетилен - воздух	2575	25.2	1.3	15.6	0.74	5.2	0.30	4.2	0.06
5 Ацетилен - кислород	3210	26.3	1.5	18.4	3.38	6.0	1.85	4.9	0.60
6 Окись углерода - кислород	2965	35.4	1.0	9.3	2.36	2.9	1.41	2.3	0.33

1) Combustible mixture; 2) hydrogen-air; 3) hydrogen-oxygen; 4) acetylene-air; 5) acetylene-oxygen; 6) carbon monoxide-oxygen.

of carbon monoxide with oxygen. The electric conductivity of combustion products with potassium additive is close to that with an additive consisting of the same fraction by weight of cesium (the difference is approximately by 1.5 times), owing to the small difference in the ionization potentials and the relatively larger concentration of the potassium particles per unit volume as compared with cesium, resulting from the much smaller molecular weight ( $\mu_K = 39.1$ ,  $\mu_{\text{Cs}} = 132.9$ ). Although the data of Table 7 have been obtained with account of the formation of negative ions, the latter can be neglected at high temperatures of the combustion products, for even under the most favorable case, at a maximum amount of the hydroxyl OH and atomic oxygen (hydrogen-oxygen and acetylene-oxygen combustible mixture) this leads only to an error that does not exceed 5% in the direction of higher electron concentration.

If we regard the combustion products of each of the combustible mixtures as a homogeneous gas having the same molecular weight as  $\mu_*$  and total cross section  $Q_*$  as the combustion products of the mixture

(i.e.,  $\mu = \mu_*$ ,  $Q = Q_*$ ), then according to calculation by means of Formulas (3.1) and (2.1) we obtain the values of the optimal additive of cesium, potassium, and sodium,  $\gamma_{\text{opt}}$ , and the corresponding maximum electric conductivity  $\sigma_{\text{max}}$  of the combustion products of the combustible mixtures with additive, as listed in Table 8. What is striking here is the high value of the optimal additive of cesium (up to 54%) for the combustion products of all the considered combustible mixtures, and also that of potassium (up to 26.7%) and sodium (up to 24%) for the combustion products of hydrogen-oxygen and hydrogen-air mixtures. For the remaining combustible mixtures, the value of the additive ranges between 2.3 and 6%, which is probably tolerable in practice, although it should reduce somewhat their combustion temperature. The value of the electric conductivity, however, does not exceed in this case  $1.85 \text{ ohm}^{-1} \text{ cm}^{-1}$  (the combustion products of the acetylene-oxygen mixture with 6% potassium), so that the burned gases, even when alkali metals are added, are unfortunately relatively poor electric conductors.

In conclusion it must be noted that the procedure of adding easily ionized substances to the working medium is essentially a method of reducing the temperature of the most difficultly ionized working medium when a definite electric conductivity is attained. Successful searches for methods of increasing the electric conductivity and further decreasing the temperature of the working medium to sensible limits without reducing the electric conductivity, along with the development of new high-temperature materials, are exceedingly important, since they would facilitate in principle the solution of many applied problems of practical importance, particularly problems connected with magnetogasdynamic conversion of heat into electricity.

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[Footnotes]

- 22 Even for such easily ionized substances as alkali metals, the error resulting from this is negligibly small and there is no sense in taking a practical account of it.
- 22 The reduction in the ionization potential [5, 7] resulting from the screening of the particle by the fields of the surrounding charges is disregarded.
- 23 The data were graciously furnished to us by L.V. Gurevich.
- 26 The ratio of the statistical weights  $2q_{O_2}/q_{O_2}$  in the calculation of the equilibrium constant of the process of formation and disintegration of negative ions of molecular oxygen is assumed to be 1.5.
- 27 The data for the hydrogen-oxygen mixture were borrowed from [26], and the data for the remaining combustible mixtures were graciously furnished by I.V. Veyts.
- 28 We disregard dissociation of molecular nitrogen.
- 35 We note that the actually measured concentration of the electrons in the flame zone itself is two to four orders of magnitude larger, depending on the composition of the combustible mixture, than that thermodynamically calculated with the Saha equation [3, 22, 36]. It is easy to see, however, that in this case the electric conductivity of the flame still remains low.

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# INFLUENCE OF THE DIFFUSION OF RADIATING IMPURITIES ON CONVECTIVE HEAT EXCHANGE

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We consider the boundary layer near the forward critical point of an axially symmetrical blunt-nosed body, from the surface of which active (radiating) gas is released. It is assumed that the boundary layer represents a binary mixture, in which no chemical reactions take place. To separate the effect of the influence of the diffusion of the radiating gas on the heat exchange, the influence of the radiation of the gas components of the main stream on the flow of the gas is neglected. Under the indicated assumptions, and using a few supplementary simplifying premises, formulas are derived for the calculation of the convective heat flows in the vicinity of the critical point of a blunted body with radiating gas released from its surface.

\* \* \*

1. In solving the problem we shall start with the results of Reference [1], in which is considered heat exchange in a frontal point over which a radiating medium flows, without any gas released from the surface of the body. Since we neglect here the influence of the radiation of the main stream on the flow of the gas in the perturbed region, unlike the cited paper, there is no need in the instance under consideration to use the complete system of Navier-Stokes equations, and it is sufficient to confine oneself to an analysis of the flow in a viscous boundary layer only. We neglect here the interaction of the vor-

tices of the shock and boundary layers, i.e., we shall consider the parameters on the outer boundary of the boundary layer to be equal to the parameters behind the departed shock wave, which enables us to use the scheme of asymptotic boundary layer [2, 3]. The latter will obviously be valid in those cases when the thickness of the boundary layer is considerably smaller than the distance  $\delta$  between the body and the shock wave [4], as will be assumed from now on.

The equations of motion and of continuity for a mixture of gases in a boundary layer, in the presence of diffusion of the radiating impurities released from the surface of the body, have the same form as in [1], and by virtue of the statements made above we must equate the terms in these equations containing the transverse pressure gradient to zero, and retain only those terms which are essential in the boundary layer.

The energy equation with account of the heat flow transported by diffusion, heat conduction, and radiation and neglecting the diffusion heat conductivity, can be represented in the following form [5]

$$\begin{aligned} \rho u \frac{\partial i}{\partial x} + \rho v \frac{\partial i}{\partial y} - \frac{\partial}{\partial y} \left( \frac{\mu}{P} \frac{\partial i}{\partial y} \right) - \mu \left( \frac{\partial u}{\partial y} \right)^2 - u_s \frac{dP_s}{dx} = \\ = \rho q_R + \frac{\partial}{\partial y} \left[ \frac{P - P_D}{P} \rho D_{12} (i_1 - i_2) \frac{\partial c_1}{\partial y} \right] \end{aligned} \quad (1.1)$$

Here  $x$  and  $y$  are the distances, measured from the forward critical point along and normal to the surface of the body, respectively;  $u$  and  $v$  are the velocity components along the direction;  $p$  is the pressure and  $q_R$  the velocity of the radiant energy heat influx; the subscripts 1 and 2 pertain to the parameters of the active gas and the main stream, respectively, while the index  $s$  pertains to the parameters behind the shock wave;  $\rho$ ,  $i$ , and  $\mu$  are the density, enthalpy, and viscosity coefficient of the mixture;  $P$  and  $P_D$  the molecular and diffusion Prandtl numbers;  $D$  is the diffusion coefficient

$$i = \sum_{j=1}^2 i_j, \quad i_j = \int_0^T c_{pj} dT + i_j^0, \quad p_D = p \frac{\lambda}{c_p^+ p D_{12}}, \quad D_{12} = D_{21}, \quad c_p^+ = \sum_j c_j c_{pj}$$

where  $i_j$  and  $c_{pj}$  are the enthalpy and the specific heat of the  $j$ -th component,  $\lambda$  the heat conduction coefficient,  $c_p^+$  the average specific heat of the mixture,  $T$  the temperature,  $i_j^0$  the production enthalpy extrapolated to absolute zero, and  $c_j$  the concentration by weight of the  $j$ -th component, satisfying the following continuity equation:

$$\rho u \frac{\partial c_j}{\partial x} + \rho v \frac{\partial c_j}{\partial y} = - \frac{\partial K_j}{\partial y} \quad (j=1,2), \quad c_1 = 1 - c_2 \quad (1.2)$$

If we neglect thermal diffusion for the binary mixture, the diffusion flux  $K_j$  contained in the right half of (1.2) is equal to

$$K_1 = -K_2 = -\rho D_{12} \frac{\partial c_1}{\partial y} \quad (1.3)$$

In the formulation of the problem as considered here, the boundary conditions for the solution of the reduced system of equations of motion, energy, and diffusion in the boundary layer will be

$$\begin{aligned} u = 0, \quad v = v_w, \quad i = i_w, \quad c_1 = c_{1w} \quad \text{for } y=0 \\ u = u_\infty, \quad i = i_\infty, \quad c_1 = 0 \quad \text{for } y \rightarrow \infty \end{aligned} \quad (1.4)$$

The subscript  $w$  denotes here the parameters on the wall.

The general expression for the rate of radiant-energy heat influx is given in [1, 6] and is not presented here in order to save space. We note only that in the case considered here the absorption coefficients of the mixture depend on the concentration  $c_1$  of the active gas, with  $\alpha_v \rightarrow 0$  as  $c_1 \rightarrow 0$ , i.e., by virtue of (1.4) we have  $q_R \rightarrow 0$  as  $y \rightarrow \infty$ .

2. Using the customary assumption made for the vicinity of the critical point, namely that terms of order  $O(x^2)$  are small, we seek the solution of the obtained system in a form analogous to the solution of [1]

$$\begin{aligned} u = f(\eta)x, \quad v = -2f(\eta) \frac{p_0}{p}, \quad i = i(\eta), \quad c_1 = c_1(\eta) \\ \frac{\partial p}{\partial x} = -\beta^2 p_0 x, \quad \beta = \frac{\sqrt{2\beta k}}{R}, \quad k = \frac{p_\infty}{p_0}, \quad \eta = \int_0^y \frac{p}{p_0} dy \end{aligned} \quad (2.1)$$

Here  $R$  is the radius of the midsection of the body,  $u_\infty$ ,  $\rho_\infty$  are the velocity and density of the incoming stream, and  $\underline{b}$  is a constant that depends on the shape of the body [7].

The problem then reduces to solution of the following system of ordinary differential equations

$$\begin{aligned} v_\infty (l/r)' + 2/r' &= f'' - \frac{P_D}{P} \\ v_\infty \left( \frac{l}{P} r' \right)' + 2/r' + c_1' \left( 1 - \frac{P_D}{P} \right) \left[ \frac{l_0 r''}{P_D} - 2/(i_1 - i_0) \right] &= -q_R \quad (2.2) \\ v_\infty \left( \frac{l}{P_D} c_1' \right)' + 2/c_1' &= 0 \end{aligned}$$

Here

$$v_\infty = \frac{P_\infty}{P_0}, \quad l = \frac{P P_0}{P_\infty P_0}, \quad c_1' = c_{11} - c_{10}$$

The third term of the left part of the energy equation in the system (2.2) takes account of the fact that when  $P_D \neq P$  the energy transport processes occur at different intensities, owing to diffusion and heat conduction. When  $P_D = P$  this equation coincides in form with the equation considered in [1]. The last simplification is used to calculate the increment of the convective heat flow, due to the radiation of the gas released from the surface of the body. In addition, we have  $l = \text{const}$ . The errors connected with the assumptions made, and the advisability of these assumptions in calculating the increment of the convective stream, will be discussed in the last section of this paper.

Under the assumptions made, the system (2.2) can be written in dimensionless coordinates in the form

$$\begin{aligned} f'' + 2/f' &= f'' - \frac{1}{P}, \quad i'' + 2/P i' = -PK^* q_R \\ c_1'' + 2/P c_1' &= 0, \quad K^* = \frac{u_\infty^2}{4i_0} \frac{1}{\sqrt{2k}}, \quad K = \frac{8\alpha_m R T_m^4}{u_\infty^2} \quad (2.3) \\ i_0 = \frac{i}{i_0}, \quad \eta_0 = \left( \frac{\beta}{i v_0} \right)^{1/2} \eta, \quad \rho_0 = \frac{P}{P_0}, \quad f_0 = (\beta l v_0)^{-1/2} f, \quad q_{R0} = (K^* \beta i_0)^{-1} q_R \end{aligned}$$

with boundary conditions

$$\begin{aligned} f = 0, f = f_0, i = i_0, c_1 = c_{10} & \text{ for } \eta = 0 \\ f \rightarrow 1, i \rightarrow 1, c_1 \rightarrow 0 & \text{ for } \eta \rightarrow \infty \end{aligned} \quad (2.4)$$

The dimensionless quantities are designated by the subscript 0, which for simplicity has been left out of the system (2.3)-(2.4); the subscript m pertains to the characteristic mean values of the corresponding quantities.

The parameter  $K^*$  characterizes the main influence of the radiation on the characteristics of the boundary layer in the vicinity of the critical point. The relation given above between  $K^*$  and the corresponding parameter  $K$ , used in [6] for a flat plate, characterizes the influence of the pressure gradient. The first factor at  $K$  is equal in practice to  $1/2$  and appears only because of some difference between the dimensionless quantities employed here and the corresponding quantities of [6]. The second factor is due to the pressure gradient in the vicinity of the critical point. With increasing pressure gradient, the velocity increases and the time that the particle stays in the vicinity of the retardation point with high values of pressure and temperature decreases, so that the influence of the radiation diminishes, i.e., the parameter  $K^*$  decreases.

We shall assume that  $q_R$  is a known quantity, determined from the profiles of the temperature and of the concentration of the active gas in the nonradiating boundary layer, the latter of which is determined by the solution of the third equation of the system (2.2)

$$c_1 = c_{10} \left( 1 - \frac{J_1(\eta)}{J_1(\infty)} \right), \quad J_1(\eta) = \int_0^\eta \exp \left( -2P \int_0^\eta f d\eta \right) d\eta \quad (2.5)$$

Here and throughout the function  $f$  is determined by the solution of the system (2.2)-(2.4) with  $K^* = 0$ .

Taking into account the radiation of the active gas released from the surface of the body, the distribution of the enthalpy of the bound-

ary layer is written in the form

$$i(\eta) = i_w + CJ_1(\eta) - PK^*J_2(\eta), \quad C = \frac{1 - i_w + PK^*J_2(\infty)}{J_1(\infty)}$$

$$J_1(\eta) = \int_0^\eta \exp\left(-2P \int_0^\eta / d\eta\right) \int_0^\eta \exp\left(2P \int_0^\eta / d\eta\right) q_R(T, c_1) d\eta d\eta \quad (2.6)$$

The heat flux transported to the surface of the body by diffusion and heat conduction is determined by the gradients of the temperature and concentration at  $\eta = 0$  in accordance with the formula

$$q_w = -\lambda \frac{\partial T}{\partial y} - \left(\rho D_{12} \frac{\partial c_1}{\partial y}\right)(i_1 - i_2) \quad (2.7)$$

Calculating the value of the derivative of the temperature on the wall and substituting it into the expression for the heat flux (2.7) we obtain

$$q_w = J_1^{-1}(\infty) P^{-1} \sqrt{\mu_s \rho_s \beta} \left\{ 1 + \frac{P i_s K^* J_2(\infty)}{\beta (i_s - i_w)} + c_{1w} \frac{i_{1w} - i_{2w}}{i_s - i_w} \right\} (i_s - i_w) \quad (2.8)$$

It was pointed out in [1] that the variability of the term containing the pressure gradient in the first equation of the system (2.3) influences the solution of the equation slightly and does not affect in practice the solution of the energy equation, so that these equations can be solved independently of each other. Moreover, Lees has shown that the right half of the equation of motion can be left out and only small errors result from this, particularly at low enthalpy factors  $i_w/i_s \ll 1$ , which is usually the situation in the cases of greatest practical interest. This enables us to employ in the numerical calculations the solution of the Blasius equation for  $f_w \neq 0$ , which has been tabulated for many values of  $f_w$  in [8]; in this case

$$J_1(\eta) = \int_0^\eta f''^2 d\eta, \quad J_2(\eta) = \int_0^\eta f''^2 \int_0^\eta f''^{-P} q_R(T, c_1) d\eta d\eta \quad (2.9)$$

3. To obtain the results in final form and for a quantitative estimate of the effect, we shall assume the active gas to be a gray radiator with a volume absorption coefficient  $k = \alpha\rho$ , which depends lin-

early on the partial pressure of the active gas,\* and we shall also neglect the reabsorption in the boundary layer. Under these assumptions, the influx rate of radiant energy can be represented in accordance with [1, 6] in the form

$$q_R = 4\sigma\epsilon_w \left[ T^4 - \frac{1}{2}(\epsilon_w T_w^4 + (2 - \epsilon_w) q_R) \right] \quad q_R = q_{R0}\sigma \quad (3.1)$$

Here  $\epsilon_w$  is the blackness coefficient of the surface about which the flow takes place,  $q_R$  is the radiant flux incident on the outer limit of the boundary layer from the high-temperature shock layer, and  $\sigma$  is the Stefan-Boltzmann constant.

Since  $\delta/R \ll 1$  for hypersonic velocities, the portion of the shock wave that radiates in the vicinity of the critical point (spherical segment) can be regarded as a plane-parallel layer with thickness  $\delta$ , and  $q_R$  can be determined from the formula

$$q_R = \int_0^\infty \pi B_\nu (1 - 2E_3(k, \delta)) d\nu, \quad E_3(\tau) = \int_0^\infty t^2 e^{-t} dt \approx \frac{1}{2} e^{-\tau} \quad (1.5 < \beta < 2) \quad (3.2)$$

Here  $B_\nu$  is the Planck function.

If we calculate  $q_R$  by using the approximate expression for the function  $E_3(\tau)$  given in (3.2), then the computation formula assumes the form

$$q_R = \epsilon_e(p, T, L_e) \sigma T^4 \quad \left( \epsilon_e = \frac{\pi}{\sigma T^4} \int_0^\infty B_\nu (1 - e^{-\beta L_e}) d\nu, \quad L_e = \beta \delta \right) \quad (3.3)$$

Here  $\epsilon_e$  is the effective blackness coefficient of a hemispherical gas volume, radiating in the center of the base of the hemisphere;  $L_e$  is the equivalent radius of the considered radiating volume. At an average value  $\beta \approx 1.8$ , as follows from (3.3), we get  $L_e = 1.8 \delta$ , which agrees with the values given in the majority of textbooks and handbooks on heat transfer [9-11] for the equivalent radii for a radiating plane-parallel layer. In the literature cited there are also plots of the



hemispherical blackness coefficients  $\epsilon_e = \epsilon_e(p, T, L_e)$  for water vapor and carbon dioxide.

The result of the calculation of  $q_r$  by Formulas (3.2) and (3.3) differ insignificantly (on the high side) from the results of the calculation which takes into account the configuration of the shock layer.\* The latter can be obtained, for example, by breaking up the spherical segment into  $n$  sectors with center at the critical point, and replacing in the  $i$ -th sector the boundary (the shock wave) by a circular arc with average radius  $r_i$ . It is easy to verify that in this case the calculation formula assumes the form

$$q_r = \sum_{i=1}^n \epsilon_i(T_i, p_i, r_i) (\sin^2 \vartheta_i - \sin^2 \vartheta_{i-1}) \sigma T_i^4 \quad (3.4)$$

Here  $\vartheta_i$  and  $\vartheta_{i-1}$  are the angles between the  $y$  axis and the radius vectors bounding the  $i$ -th sector.

Formula (3.4) enables us to take into account the distribution of the parameters along the length of the radiating segment. The maximum deviation of the parameters in the segment from their values in the vicinity of the retardation point will obviously be on the line where the shock wave crosses the plane tangent at the critical point to the body around which the stream flows. It is easy to show, by using the relations on the discontinuity, that these deviations are insignificant and, for example, for a hemispherical nose they turn out to be small quantities of the order  $\delta/R \approx k$ , i.e., the parameters in the radiating volume change little, so that we can use Formulas (3.2) and (3.3) to calculate  $q_r$ .

In the absence of absorbing gas in the boundary layer, the flux  $q_r$  calculated in this manner is added to the convective flux. On the other hand, in the case considered here, the flux  $q_r$ , as follows from (2.8) and (3.1), acts on the convective flux and the additivity is de-

destroyed.

Under the assumptions of the present section, Expression (3.1), using the solution (2.5) with  $P = 1$  and the relations

$$p_1 = p n_1, \quad n_1 = \frac{m_2 p_1}{c_1 (m_2 - m_1) + m_1}, \quad c_{1w} = \frac{-2j_w}{-2j_w + j_w^2}, \quad k = G p_1$$

$$p = p_2 \left[ 1 + \left( \frac{m_2}{m_1} - 1 \right) c_1 \right]^{-1}, \quad i = i_w + (i_0 - i_w) f, \quad c_p^* = c_{p1} c_1 + c_{p2} (1 - c_1)$$

can be represented in the following form

$$q_R = - \frac{8\pi G p m_2 j_w (1-f) [i_w + (i_0 - i_w) f]}{p_2 i_0 (j_w^2 - 2j_w) m_1} \left\{ \left[ \frac{i_w + (i_0 - i_w) f}{c_{p1} c_{1w} + c_{p2} (1 - c_{1w})} \right]^2 - B \right\}$$

$$B = \frac{1}{2} (c_w T_w^2 + (2 - c_w) q_{w0}) \quad (3.5)$$

Here  $m_1$  and  $m_2$  are the molecular weights of the active gas and of the main stream, respectively;  $G$  is a proportionality coefficient that depends in the general case on the total pressure in the mixture and on the local value of the temperature. Assuming

$$c_{p1} \approx c_{p2} = \text{const}, \quad m_1 \approx m_2, \quad G(p, T) = G(p)$$

we obtain from (2.8), (2.9), and (3.5) after simple transformations

$$q_w = q_{w0} \left\{ 1 - K_0 \sum_{n=0}^{\infty} C_n \left( \frac{T_w}{T_0} - 1 \right)^n \left( \frac{i_w}{i_0 - i_w} I_n + I_{n+1} \right) + K_0 B \left( \frac{i_w}{i_0 - i_w} I_0 + I_1 \right) \right\} \quad (3.6)$$

$$K_0 = \frac{8\pi G p i_0 T_w^2}{\beta U_w^2 - 2j_w p_2 i_0}, \quad I_n = \int_0^1 f^n \int_0^1 \frac{1-f}{f} d\eta d\eta$$

Here  $q_{w0}$  is the convective heat flux calculated neglecting the radiation of the gas released from the surface of the body, and  $C_m^n$  is the number of combinations of  $m$  taken  $n$  at a time. The integrals  $I_n$  are universal functions that depend on the intensity of the in-blast of the active gas; let us present the values of  $I_0, I_1, \dots, I_5$  for three values of  $f_w$ .

$I_w$	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
-0.500	10.975	1.0648	0.3011	0.1394	0.08255	0.05578
-0.375	3.8598	0.6485	0.2333	0.1194	0.07392	0.05137
-0.250	2.0310	0.3036	0.0966	0.0446	0.02528	0.01639

4. Formula (3.6), which was obtained under the numerous simplifying assumptions of the preceding section, illustrates the qualitative influence of the diffusion of the radiating impurities on convective heat exchange. It follows from (3.6) that the presence of radiating and absorbing impurities in the boundary layer can lead to two opposing effects. The second term of (3.6) determines the effects of deexcitation of the boundary layer, due to the energy loss resulting from the radiation of the active gas. This effect leads in accordance with (3.6) to a reduction in the convective heat exchange. The third term is due to absorption by the active gas of energy radiated by the shock layer and by the surface of the body in the stream. Unlike the deexcitation effect, the effect of reabsorption leads, in accordance with (3.6), to an increase in the convective fluxes. It is obvious that this effect is determined by the magnitude of the radiant flux  $q_r$ , radiated by the shock layer. However, the influence of the radiation of the shock layer is not limited to the foregoing effect. In addition, the flow in the boundary layer can also be changed as a result of a change in the gas parameters on its outer boundary due to the deexcitation of the shock layer, something not taken into account in the preceding sections. In order to take this effect into account, it is sufficient to substitute in the formulas obtained above the value of the enthalpy on the outer boundary, calculated with account of the deexcitation of the shock layer. In this case we can neglect the deexcitation of the boundary layer resulting from the radiation of the particles of the main stream (for example, air), by virtue of the relatively low temperatures in the boundary layer as compared with the temperature of the shock layer  $[(T_w/T_s)^4 \ll 1]$ , i.e., the deexcitation of the boundary layer is due only to the radiation of the active gas.

The distribution of the enthalpy between the outer limit of the

boundary layer and the shock wave, due to the deexcitation of the shock layer, can be readily determined by using the method indicated in [1], namely: we put in the equations of the cited paper  $\lambda = \mu = 0$ . Then the distribution of the velocities in the shock layer is determined by the solution of Reference [12], while the temperature profile is given by the equation

$$\begin{aligned} & (v^2/2 \ll l_0, \quad (T_0/T_\infty)^2 \ll 1) \\ & \int \frac{dT}{dy} = -2\alpha T^2, \quad \alpha = \int_0^\infty \alpha_p p_p dv, \quad p_0 = \frac{\pi B_0}{\sigma T^4} \quad (4.1) \\ & I = \left( \frac{kv_\infty}{2} - \beta\delta \right) \left( \frac{y}{\delta} \right)^2 + \beta y, \quad T(\delta) = T_0 \end{aligned}$$

In the first equation of (4.1) we can assume  $c_p$  and  $\alpha$  constant, equal to some values averaged over the thickness of the shock layer [13]. In this case, as was indicated in [1] the temperature profile in the shock layer is determined in final form by

$$T = T_0 \left( 1 + K_0 \ln \frac{1}{y} \frac{\delta + Ny}{1+N} \right), \quad K_0 = \frac{6\sigma RT_0^2}{c_p u_\infty \sqrt{2k}}, \quad N = \frac{Rk - 2\sqrt{2k}\delta}{2\sqrt{2k}\delta} \quad (4.2)$$

For a hemispherical small nose we have  $\delta/R = k(1 + \sqrt{2k})^{-1}$ , and consequently

$$N = (1 - \sqrt{2k})/2\sqrt{2k}$$

By a method that differs somewhat from that indicated in [1], this solution was obtained in [14]. It differs from the solution (4.2) only in the fact that the parameter  $K_0$  contains a factor  $E_2(0.5\tau_\delta)$ , which is close to unity and takes approximately into account the reabsorption in the shock layer\* ( $\tau_\delta$  is the optical thickness of the shock layer). The solution obtained has a singularity at  $y \rightarrow 0$ , but in the case under consideration it can be used to determine the parameters on the outer limit of the boundary layer, parameters contained in Formulas (2.8) and (3.6). This singularity was considered previously in References [16, 1, 14]. In connection with the foregoing singularity, and with the need for using a finite thickness of boundary layer when tak-

ing into account the deexcitation of the shock layer, it must be noted that in the vicinity of the critical point, by virtue of the large vorticity of the stream in the nonviscous part of the shock layer, the solution of the radiating boundary layer should go over into the solution for a radiating shock layer, determined for the same value of the stream line. At the same time, as in flat bodies, the parameters on the outer limit of the boundary layer are equated to the parameters on the zero stream line of the nonviscous flow, obtained without account of the boundary layer [4], which makes it possible in many cases of flow around flat bodies to carry out an asymptotic transition from the solution for the radiating boundary layer into the solution of the problem for the nonviscous radiating shock layer [18].

It is obvious that in calculating the radiant flux  $q_r$  from a shock layer with allowance for its deexcitation, it becomes necessary to take into account the variation of the parameters over the thickness of the radiating segment. In this case  $q_r$  can be calculated by breaking up the  $i$ -th sector in (3.4) into  $m_i$  concentric layers, summing then the radiant flux from all  $m_i$  elements with allowance for their attenuation in the preceding layers. For the case of gray radiation, the calculation formula can be represented in this case in the form

$$q_r = \sum_{i,j=1}^{n,m_i} \epsilon_{ij} \prod_{l=0}^{j-1} (1 - \epsilon_{il}) (\sin^2 \theta_i - \sin^2 \theta_{i-l}) \sigma T_{ij}^4 \quad (4.3)$$

Here  $\epsilon_{ij} = \epsilon_e(r_{ij} - r_{i,j-1}, p_{ij}, T_{ij})$  is the effective blackness coefficient of a hemispherical gas volume with radius equal to the thickness of the  $ij$ -th element,  $r_{ij}$  is the radius vector of the  $ij$ -th element, while  $T_{ij}$  and  $p_{ij}$  are the temperature and pressure in the  $ij$ -th element.

Thus, the summary effect connected with the presence of the radiation field in the gas stream, is brought about by the deexcitation of

the radiating impurities of the active gas in the boundary layer, leading to a decrease in the convective streams, by the absorption of radiant flux  $q_r$  from the shock layer by the impurities, leading to an increase in the convective streams, and by the deexcitation of the shock layer, which leads to a reduction in  $q_r$  and in the temperature on the outer limit of the boundary layer. In addition, the active gas screens somewhat the surface of the body against the radiant flux  $q_r$ , i.e., the fraction of the energy  $q_r$  absorbed by the gas is not offset, owing to the relatively low temperatures in the boundary layer, by its radiation (it is obvious that Formula (4.3) takes into account this screening effect).

Consequently, the absorption of the active gas, which leads (by deformation of the temperature profile in the boundary layer) to an increase in the convective flux, reduces somewhat the radiant flux. On the other hand the deexcitation, without causing an appreciable increase in the radiant flux to the body  $q_r$ , can noticeably attenuate the profile of the temperature in the boundary layer and accordingly decrease the convective flux.

Depending on which of the foregoing effects plays the predominant role, the action of the radiation field on the temperature field in the perturbed region can lead to either an increase or to a decrease in the summary flux to the shell of the body in the stream:

$$q_z = q_{\text{conv}} + A_w q_r - \epsilon_w \sigma T_w^4 \quad (4.4)$$

where  $A_w$  is the absorbing ability of the surface of the body (it is frequently assumed that  $A_w = \epsilon_w$ , in accordance with Kirchhoff's integral law), and  $z$  is a factor that takes into account the influence of the radiation field on the convective heat exchange.

In the case under consideration, the factor  $z$  is determined by Formulas (2.8) or, with a smaller degree of accuracy, by (3.6). These

formulas obviously can be used not only for a qualitative analysis of the influence of the diffusion of the radiating impurities on the heat exchange, but also for numerical calculations.

Indeed, the representation of  $q_w$  in the form  $q_{w0}z$  is quite convenient, and if  $q_{w0}$  is determined with allowance for the main properties of the real gas, then the integral quantities  $z$  can be calculated with sufficient accuracy under the assumptions made in the second and even in the preceding sections. Obviously, the variability of  $l$ , the deviation of the Lewis number from unity, or an exact account of the terms containing the pressure gradient, etc. can introduce only small corrections into  $z$ , and cannot change appreciably the value of the thermal flux, all the more since all these factors are taken into account by the first (main) factor of the formulas obtained.

From these formulas we can readily establish also the influence of the blackness coefficient of the surface  $\epsilon_w$  on the increment of the convective flux due to the radiation. The decrease in  $\epsilon_w$  as a function of the temperature factor  $T_w/T_s$  can lead both to a decrease or to an increase in the convective heat fluxes.

If we neglect reabsorption for gray radiation with  $T_w/T_s < \epsilon_e^{0.25}$  the decrease in  $\epsilon_w$ , as follows from (3.6) and (3.3), leads to an increase in the convective fluxes, while in the case of  $T_w/T_s > \epsilon_e^{0.25}$  it leads to their decrease. The meaning of these inequalities is quite obvious. For example, when  $T_w \approx 0$  the first inequality is always satisfied, and leads to an increase in the fluxes with decreasing  $\epsilon_w$ . In this case when  $\epsilon_w = 0$  the increment of the flux is approximately doubled as compared with  $\epsilon_w = 1$ . The latter is due to the fact that in the former case the boundary layer is traversed twice by the radiant flux (incident from the shock layer and reflected from the surface of the body), whereas when  $\epsilon_w = 1$  the radiant flux passing through the

boundary layer is completely absorbed by the wall.

It is obvious that with increasing  $T_w$  and with decreasing  $\epsilon_e$ , the principal role will be assumed by the absorption of the thermal radiation of the body by the active gas, and consequently if the second of the foregoing inequalities is satisfied an increase in  $\epsilon_w$  leads to an increase in the convective thermal fluxes.

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[Footnotes]

- 51 We note that the use of a more general relationship does not add additional difficulties to the solution.
- 52 This difference decreases with increasing optical thickness of the shock layer and consequently with increasing  $q_r$ .
- 55 According to [6], if reabsorption is taken into account, the right half of (4.1) assumes in the first approximation the form  $-\beta\sigma\alpha T^4$  and only when  $\tau_\delta \approx 0$  does it go over into  $-2\sigma\alpha T^4$ , since  $\beta(0) = 2$ , where  $\beta$  is determined from (3.2). It follows therefore that for an approximate account of reabsorption it is sufficient to multiply the parameter  $K_\delta$  in the solution (4.2) by  $\beta/2$ . It is obvious that this factor has the same meaning as  $E_2(0.5 \tau_\delta)$ , and when  $\tau_\delta \approx 0$  it also turns into unity. We note that both methods of taking reabsorption into account can be used only for relatively small  $\tau_\delta$ . Large optical thicknesses are considered in [15-17].



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EFFECT OF RATE OF HEAT RELEASE IN ELECTRIC EXPLOSIONS IN WATER  
ON THE DISTRIBUTION OF THE EXPLOSION ENERGY

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In the case of explosions in water it can be assumed that the energy is essentially divided between the energy carried away by the shock wave and the energy of radial motion of the layers of the medium adjacent to the gas bubble.

We investigate in this work the energy distribution of an underwater electric explosion among the above-mentioned types of motion, as a function of the initial velocity of energy release. The initial rate of energy release was varied by connecting different inductances in the discharge circuit.

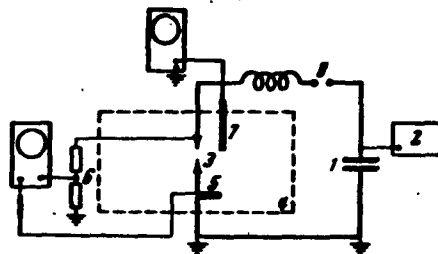


Fig. 1

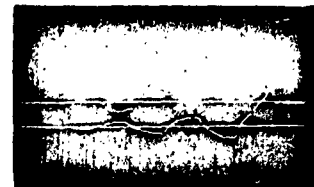


Fig. 2

An attempt to determine the ratio of the energy of the positive phase of the acoustic wave in water in the electric explosion to the total energy stored in the capacitor, for different parameters of the discharge circuits, was made in [1]. The dependence of the radiated sound energy on the total discharge energy, the inductance, and the

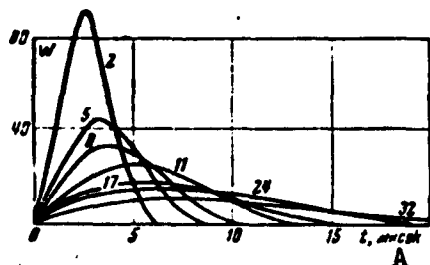


Fig. 3. A)  $\mu\text{sec}$ .

resistance is described in the article [2].

Description of the setup. The diagram of the setup is shown in Fig. 1. A capacitor bank 1 with total capacitance  $1.35 \mu\text{f}$ , charged with rectifier 2 to a

voltage of 20 kv, was discharged through a spark gap 3 20 mm in length in a bath 4 at a depth of 140 mm. The bath measured  $500 \times 500 \times 1000 \text{ mm}^3$  and was filled with tap water. The inductance of the circuit was varied between 2 and  $32 \mu\text{h}$ . The time dependence of the current and voltage on the spark gap was recorded with an OK-25 oscillograph. The current transducer was a noninductive shunt 5 made of nichrome ribbon with divider; the voltage transducer was divider 6, made up of type VS resistors without threading.

The pressure wave from the spark was registered with an EO-58M oscillograph. The pressure transducer 7 was a cylinder 7 mm in diameter and 10 mm high, made of polarized barium titanate ceramic, soldered with Wood's alloy to a long zinc rod of the same diameter. This system was placed in an aluminum tube of inside diameter 10 mm and the gap filled with wax. The pressure transducer was calibrated in a shock tube. The time resolution was on the order of  $5 \mu\text{sec}$ .

Simultaneously with registering the current, the voltage, and the pressure, we photographed the first pulsation of the gas bubble on the rotating disk of the photoregister. The illumination was by means of an IFP-1500 gas-discharge flash lamp through a window in the bath.

Dependence of the energy of the positive phase of the pressure wave on the rate of energy release. Figure 2 shows a time sweep of the current and the voltage of the spark gap. The oscillograph was triggered at the instant when the air gap 8 was closed (Fig. 1), when the

voltage applied to the spark gap in the water is 20 kv. However, the breakdown of the spark gap in the water is not instantaneous, but after a certain delay, during which the spark channel is formed. The delay time varies and reaches 10-15  $\mu\text{sec}$ . The discharge has the character of damped harmonic oscillations; the period and the damping decrement depend on the parameters of the discharge circuit.

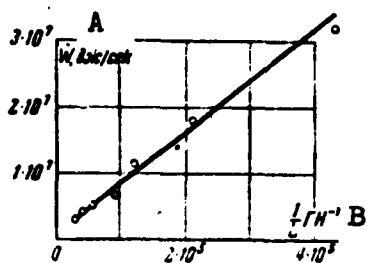


Fig. 4. A)  $\dot{W}$ , joule/sec;  
B)  $1/L$ ,  $\text{h}^{-1}$ .

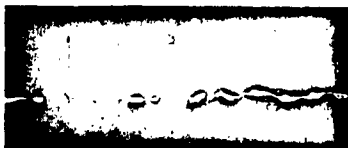


Fig. 5

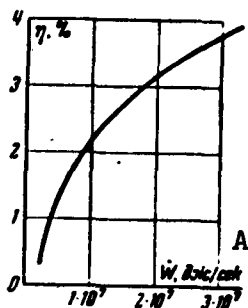


Fig. 6. A)  $\dot{W}$ , joule/sec. ductance  $L$  of the discharge circuit are indicated on the curves. From the initial slope we determine the initial rate of energy release  $W$ .

For each of the seven values of the inductance we made 20 experiments, two with average delay time were in error.

Experiment shows that the initial rate of energy release was in-

The current and voltage oscillograms make it possible to calculate the following:

1. The total energy released in the spark gap, given by

$$\int_0^{\infty} UI dt$$

Experiment has shown that the value of the integral varies little with the inductance in the circuit and is approximately equal to  $CU^2/2$ , i.e., all the energy stored initially in the capacitor. In the experiments  $W_0 = 270$  joules.

2. Using the oscillograms, we can plot the dependence of  $W(t)$  in joules of released energy on the time, as shown in Fig. 3, where different values of the in-

versely proportional (Fig. 4) to the inductance of the discharge circuit,  $L$ .

The energy  $W_p$  of the positive phase of the pressure wave is an important characteristic of the underwater explosion and serves as a measure of the useful work which can be performed by the shock wave. Assuming spherical symmetry, the work  $W(R)$  on a surface of radius  $R$ , performed after the arrival at the wave front at the instant  $t = 0$  is determined by the expression

$$W(R) = 4\pi \int_0^{\tau} R P U dt$$

where  $U$  is the stream velocity on a surface of radius  $R$  and  $U dt$  is the displacement of the surface during the time  $dt$ .

If it is required to calculate on the basis of the experimental pressure-time curves (the oscillogram on Fig. 5) the energy radiated in a pressure wave of finite amplitude, then the stream velocity can be eliminated by means of the expression

$$U = (p - p_0) / \rho_0 c_0$$

As a result, neglecting the hydrostatic pressure  $p_0$ , we obtain

$$W(R) = S \frac{1}{\rho_0 c_0} \int_0^{\tau} p^2 dt$$

Here  $S$  is the area of the sphere whose radius is equal to the distance from the spark gap to the pressure transducer;  $\rho_0 c_0$  is the wave resistance of the water;  $\tau$  is the duration of the positive phase of the pressure wave, and  $p$  is the peak pressure [3].

Experiment has shown that the duration of the positive phase of the pressure wave,  $\Delta t_+$ , does not depend on the rate of energy release in the spark gap. The quantity  $\Delta t_+$  is a function of the initial radius of the gas bubble, whose dimensions are determined by the distance between the electrodes of the spark gap [4].

At given parameters of the discharge circuit, the energy of the pressure wave at a distance of 20 cm from the source amounts to only a few per cent of the energy initially stored in the capacitor.

The dependence of  $\eta$  (the ratio of the energy of the positive phase of the pressure wave to the energy initially stored in the capacitor) on the initial rate of energy release in the spark gap  $W$  is shown in Fig. 6.

The energy of the radial motion of the liquid in the gas bubble can be expressed as the potential energy of the expanding bubble in terms of its maximum radius  $a$  and the hydrostatic pressure  $p_0$  at the explosion depth using the formula

$$Y = \frac{1}{3} \pi a^3 p_0$$

The maximum radius of the bubble is determined from the photographs, one of which is shown in Fig. 7 (time scan of the expansion and compression of the gas bubble, arising during the electric discharge in the water). The photograph was taken through a slit by scan-

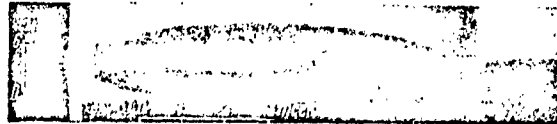


Fig. 7. msec = msec.

ning on the rotating disk of the photoregister. The maximum radius of the bubble is independent, within the limits of experimental error, of the rate of energy release. The result is understandable, since the total amount of energy does not change, and the energy-release process always remains sufficiently rapid compared with the expansion of the gas cavity. The energy of radial motion of the liquid layers adjacent to the gas bubble amount to about 22% of the energy initially stored in the capacitor, and is independent of the initial rate of energy re-



lease.

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